Data-driven verification and control moving from linear dynamical systems to nonlinear stochastic systems

9TH OF JULY 2023

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TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Data-driven Verification and Correct-by-Design Control

This talk:

How can you deal with the lack of environment and model knowledge in safety critical applications?

1. Direct data-drivencontrol for linear dynamical systems

2. Approximate simulation relations for nonlinear stochastic systems with model uncertainty

3. Embedding environment uncertainty

[Amazon]

[TheVerge]

[Cda]

[BBC]

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Contents

Direct data-driven temporal logic control

True & unknown LTI system dynamics:

$x(t+1) = Ax(t) + Bu(t)$

$y(t) = Cx(t)$

Specification in Signal Temporal Logic:

$$\varphi ::= \sigma \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \square_{[a,b]}\varphi \mid \varphi_1 \, \mathsf{U}_{[a,b]} \, \varphi_2, \qquad (1)$$

Given a single finite data trace $w_{data} = (u_{data}, y_{data})$ design $u = \{u_0, u_1, y_2 \dots\}$ such that $y = \{y_0, y_1, y_2, \dots\}$ satisfies φ.

van Huijgevoort, B. C., Verhoek, C., Tóth, R., & Haesaert, S. (2023). Direct data-driven signal temporal logic control of linear systems. *arXiv preprint arXiv:2304.02297*.

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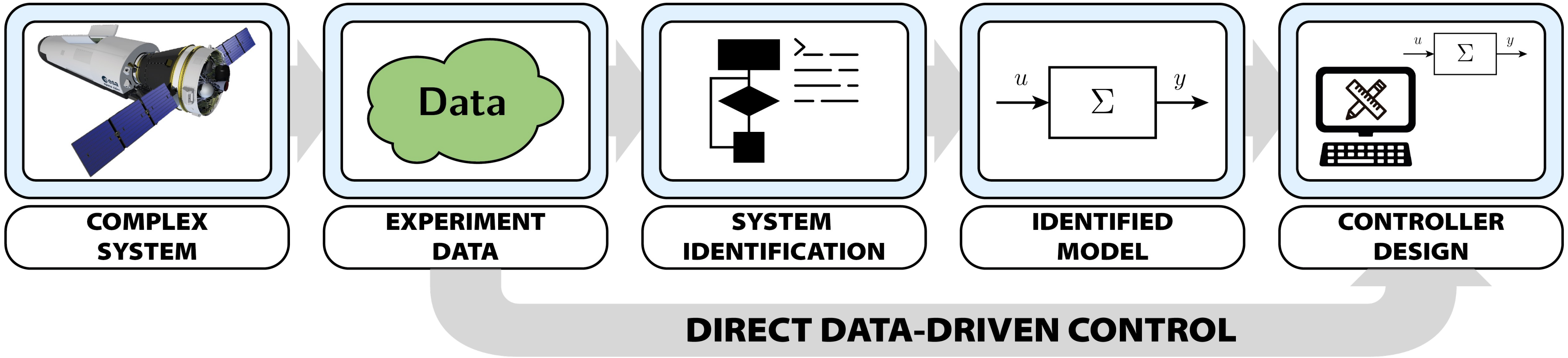
Direct data-driven control: The idea

Achieve controller design directly from data



- Predictive control schemes (e.g., DeePC [3])
- State-feedback control [4]
- Dissipativity analysis [5]
- Noise handling & robustness guarantees [6]

[3] Coulson, Lygeros, Dorfler (2019) “Data-Enabled Predictive Control: In the Shallows of the DeePC,” in *Proc. of the 2019 ECC*.
[4] Markovsky & Rapisarda (2008) “Data-driven simulation and control,” *Int. Journal of Control*.
[5] Romer et al. (2019) “One-shot verification of dissipativity properties from input-output data,” *Control Systems Letters*.
[6] Berberich et al. (2021) “Data-Driven Model Predictive Control With Stability and Robustness Guarantees,” IEEE-TAC

Direct data-driven temporal logic control

Model-based control STL

$$\arg\min_{\boldsymbol{u}} \ J(\boldsymbol{u}, \boldsymbol{y})$$
$$s.t. \quad x(t+1) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

$$\boldsymbol{y_0} = [y(0), y(1), y(2), \dots]$$
$$\boldsymbol{y_0} \vDash \boldsymbol{\varphi}$$

Model equations

Specification in Signal temporal logics can be written as a MILP

Donzé, Alexandre, et al. "BluSTL: Controller Synthesis from Signal Temporal Logic Specifications." *(2015)*

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Direct data-driven temporal logic control

$$\arg\min_{\boldsymbol{u}} \; J(\boldsymbol{u}, \boldsymbol{y})$$

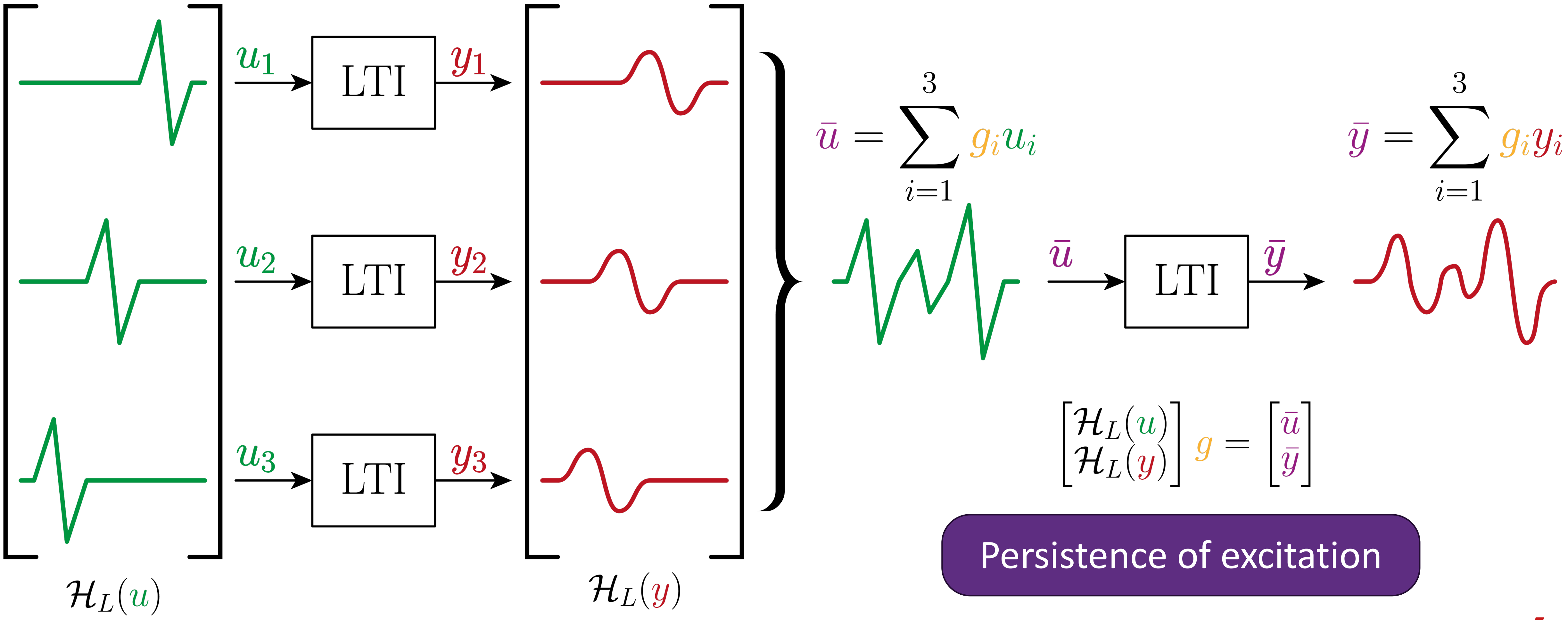
$s.t.$ Willems fundamental lemma ⟸ Replace model with data

$$\boldsymbol{y}_0 = [y(0), y(1), y(2), \dots]$$
$$\boldsymbol{y}_0 \vDash \boldsymbol{\varphi}$$

van Huijgevoort, B. C., Verhoek, C., Tóth, R., & Haesaert, S. (2023). Direct data-driven signal temporal logic control of linear systems. *arXiv preprint arXiv:2304.02297*.

Willems’ Fundamental Lemma (main idea)

A behavioral approach to data-driven modeling



[1] Polderman, Willems (1998) ”Introduction to Mathematical Systems Theory: A Behavioral Approach” Springer
[2] Willems et al. (2005) “A note on persistency of excitation” *Systems & Control Letters*

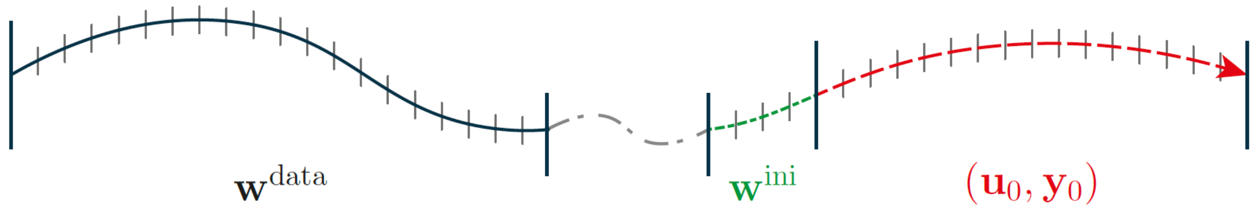
Direct data-driven temporal logic control

$$\arg\min_{\boldsymbol{u}} \; J(\boldsymbol{u}, \boldsymbol{y})$$

$s.t.$ Willems fundamental lemma

Replace model with data

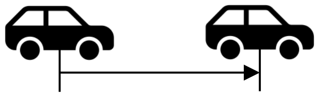
$$\boldsymbol{y}_0 = [y(0), y(1), y(2), \dots]$$
$$\boldsymbol{y}_0 \vDash \boldsymbol{\varphi}$$



van Huijgevoort, B. C., Verhoek, C., Tóth, R., & Haesaert, S. (2023). Direct data-driven signal temporal logic control of linear systems. *arXiv preprint arXiv:2304.02297*.

Direct data-driven temporal logic control

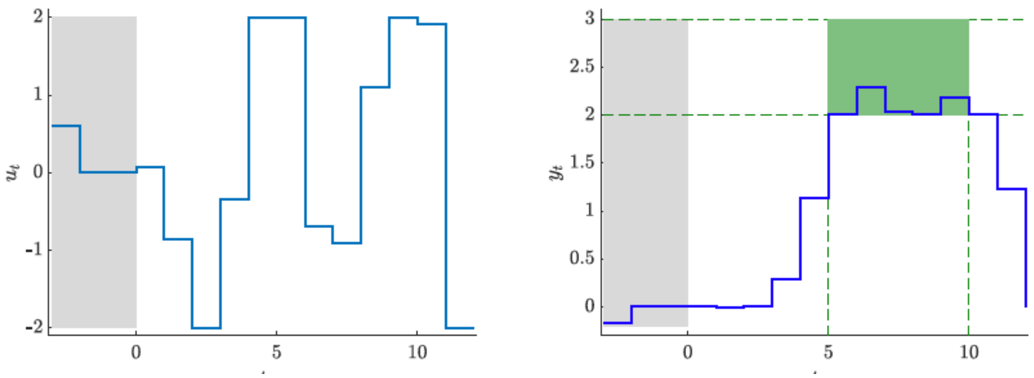
Car platoon case study



$$\varphi = \square_{[5,10]}(|y| \geq 2 \wedge |y| \leq 3).$$

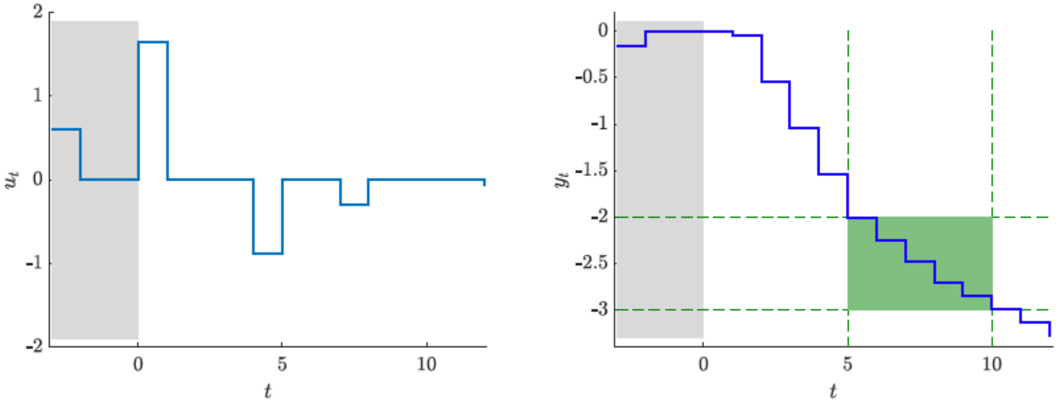
Minimize output & satisfy φ

$$J_1(\mathbf{u}_0, \mathbf{y}_0) = \|\mathbf{y}_{[0,L]}\|$$



Minimize output & satisfy φ

$$J_2(\mathbf{u}_0, \mathbf{y}_0) = \|\mathbf{u}_{[0,L]}\|$$



Direct data-driven temporal logic control

$$\arg\min_{\boldsymbol{u}} \; J(\boldsymbol{u}, \boldsymbol{y})$$

$$s.t. \quad \text{Willems fundamental lemma}$$

$$\boldsymbol{y}_0 = [y(0), y(1), y(2), \ldots]$$
$$\boldsymbol{y}_0 \vDash \boldsymbol{\varphi}$$

Based on the fundamental Lemma

- Sound for LTI systems
  (based on Willems fundamental lemma)
- Completeness based on persistence of excitation

van Huijgevoort, B. C., Verhoek, C., Tóth, R., & Haesaert, S. (2023). Direct data-driven signal temporal logic control of linear systems. *arXiv preprint arXiv:2304.02297*.

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Applicability of fundamental lemma

Excellent results:

- Predictive control schemes (e.g., DeePC [3])
- State-feedback control [4]
- Dissipativity analysis [5]
- Noise handling & robustness guarantees [6]

$$\begin{bmatrix}\mathcal{H}_L(u^{\mathrm{d}})\\ \mathcal{H}_L(y^{\mathrm{d}})\end{bmatrix} g = \begin{bmatrix}\mathrm{col}(\bar{u})\\ \mathrm{col}(\bar{y})\end{bmatrix}$$

Generalizations:

- Linear Time-Varying (LTV), bilinear systems, stochastic linear systems [7-9]
- Wiener & Hammerstein, flat nonlinear, NARX systems [10-12]
- **Linear Parameter-Varying Systems** [13]

[7] Nortmann & Mylvaganam (2020) “Data-Driven Control of Linear Time-Varying Systems,” in *Proc. of the CDC 2020*.

[8] Yuan & Cortés (2022) “Data-driven optimal control of bilinear systems,” *IEEE Control Systems Letters*.

[9] Faulwasser et al. (2023) “Behavioral theory for stochastic systems? A data-driven journey from Willems to Wiener and back again,” *Ann. Rev. in Control*.

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[12] Mishra et al. (2021) “Data-driven simulation for NARX systems,” in *Proc. of the ESPC 2021*.

[13] Verhoek, Tóth, Haesaert, Koch (2021). "Fundamental Lemma for Data-Driven Analysis of Linear Parameter-Varying Systems." In *Proc. of the CDC 2021*.

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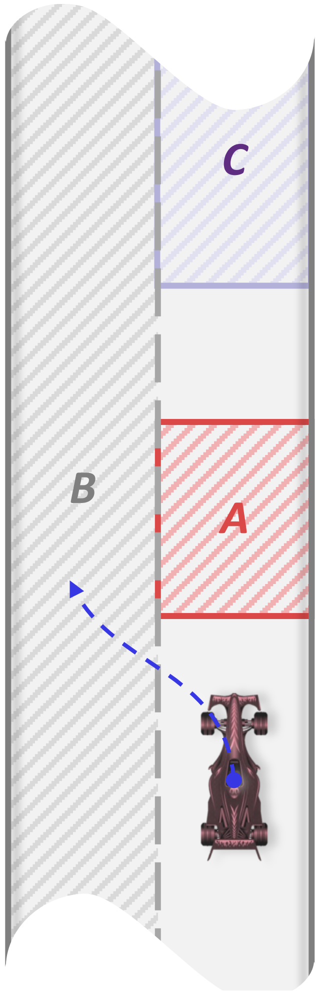
Verification and Correct-by-Design Control for Stochastic Systems



Stochastic system: $M: \{ x_{k+} \ldots x_k, u_k) + w_k$ ... $_k = h($ $\quad w_k \sim p_w(\cdot)$

Specification: $\psi = (\neg \boldsymbol{A} \text{ U } \boldsymbol{C}) \wedge \bigcirc(\boldsymbol{B} \rightarrow \Diamond \boldsymbol{C})$

LTL semantics: $\psi ::= p \mid \neg\psi \mid \psi_1 \vee \psi_2 \mid \psi_1 \wedge \psi_2 \mid \psi_1 \text{U} \psi_2 \mid \bigcirc\psi$

Task: Design controller such that controlled system exhibits specified behavior ψ with probability at least p_ψ.

✓ **Solvable for known continuous stochastic systems**

S. Haesaert & S. Soudjani (2020) *Robust dynamic programming for temporal logic control of stochastic systems. TAC.*

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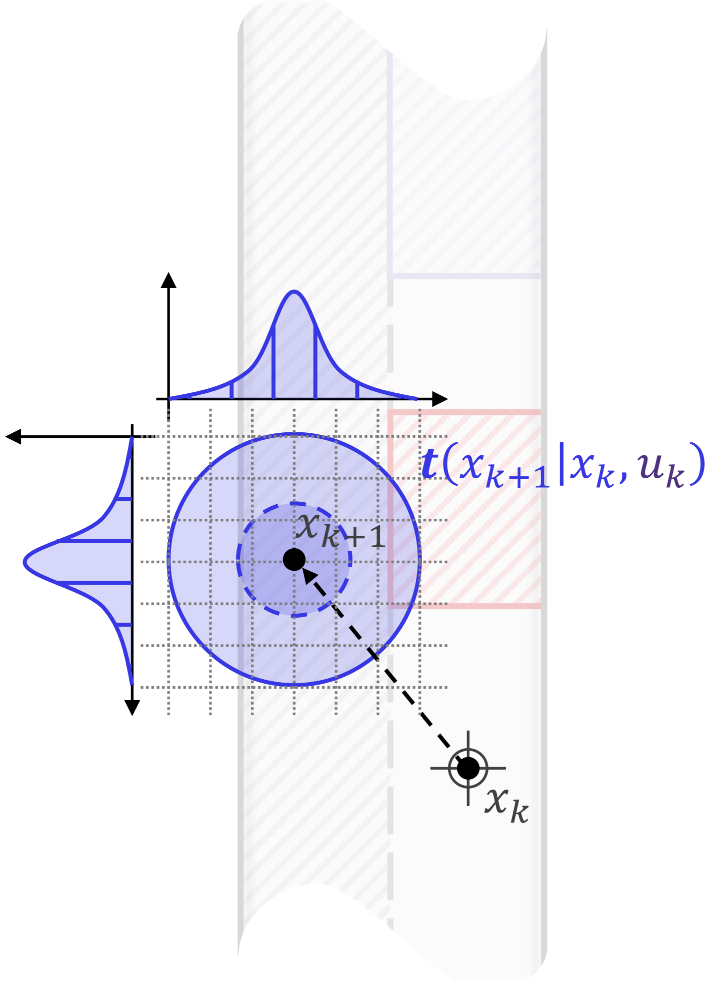
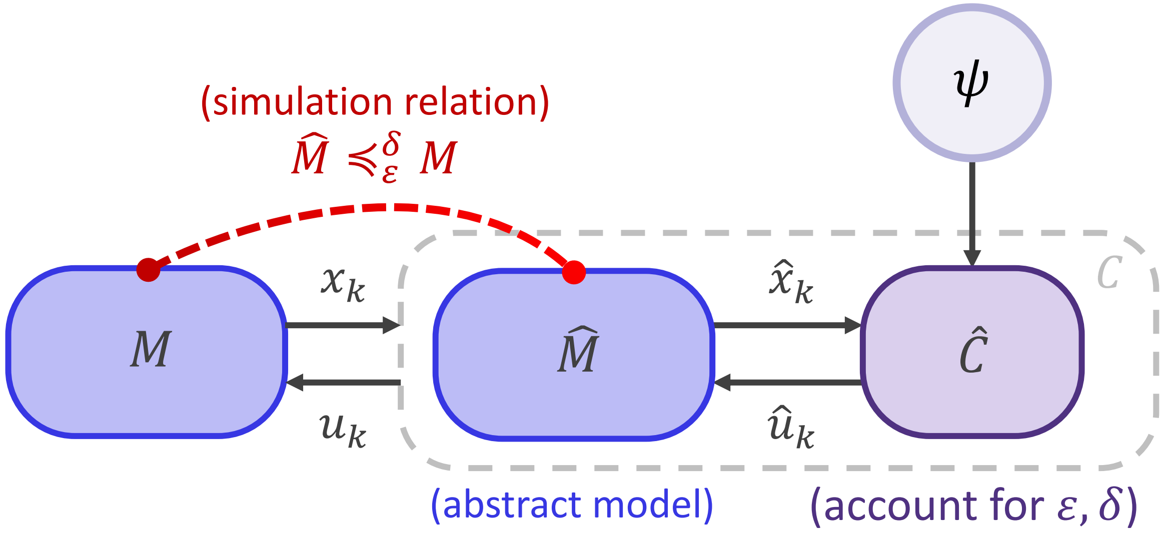
Correct-by-Design Control (Full Knowledge)

Abstraction-based solution





If ① $\widehat{M} \preccurlyeq_{\varepsilon}^{\delta} M$, then ② $\mathbb{P}\{C \times M \vDash \psi\} \geq \mathbb{P}\{\hat{C} \times \widehat{M} \vDash \psi\} \geq p_{\psi}$

S. Haesaert & S. Soudjani (2020) *Robust dynamic programming for temporal logic control of stochastic systems. TAC.*

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Toolbox “SySCoRe”

Temporal logic control for stochastic systems

- **Input:** Stochastic system model + specification
- **Output:** Robust policy + bounds on satisfaction probability

Features:

- ✓ Nonlinear stochastic dynamics
- ✓ (Arbitrary) unbounded support
- ✓ (Possibly) infinite horizon
- ✓ Unbounded scLTL specifications
- ✓ Model order reduction supported
- ✓ Exploits fast tensor computations

Toolbox: SySCoRe (v1 available)

https://github.com/BirgitVanHuijgevoort/SySCoRe-software

Tool paper: B.C v. Huijgevoort et al. (2023) *SySCoRe: Synthesis via Stochastic Coupling Relations. HSCC’23.*

Friendly competition: *ARCH-COMP22 Category Report: Stochastic Models*. EPiC.

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Correct-by-Design Control of Uncertain Systems

True system:

$$M: \begin{cases} x_{k+1} = {\theta^*}^T f(x_k, u_k) + w_k \\ y_k = h(x_k) \end{cases} \quad w_k \sim p_w(\cdot)$$

Set of models:

$$M: \begin{cases} x_{k+1} = \theta^T f(x_k, u_k) + w_k, \text{for all } \theta \in \Theta \\ y_k = h(x_k) \end{cases} \quad w_k \sim p_w(\cdot)$$

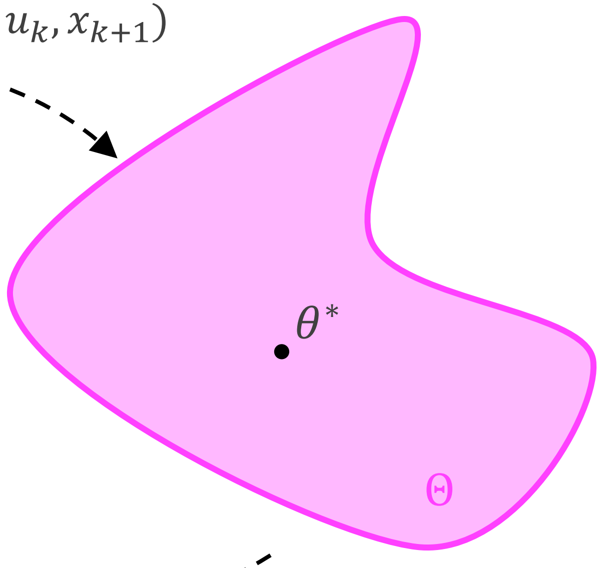
Bayesian guarantees:

$$\mathbb{P}\{\theta^* \in \Theta\} \geq (1 - \alpha)$$

or equivalently

$$\mathbb{P}\{M(\theta^*) \in \{M(\theta) \mid \theta \in \Theta\}\} \geq (1 - \alpha)$$

sample (x_k, u_k, x_{k+1})



(credible set Θ)

translate θ

O. Schön et al. (2023) *Bayesian Approach to Temporal Logic Control of Uncertain Systems.*

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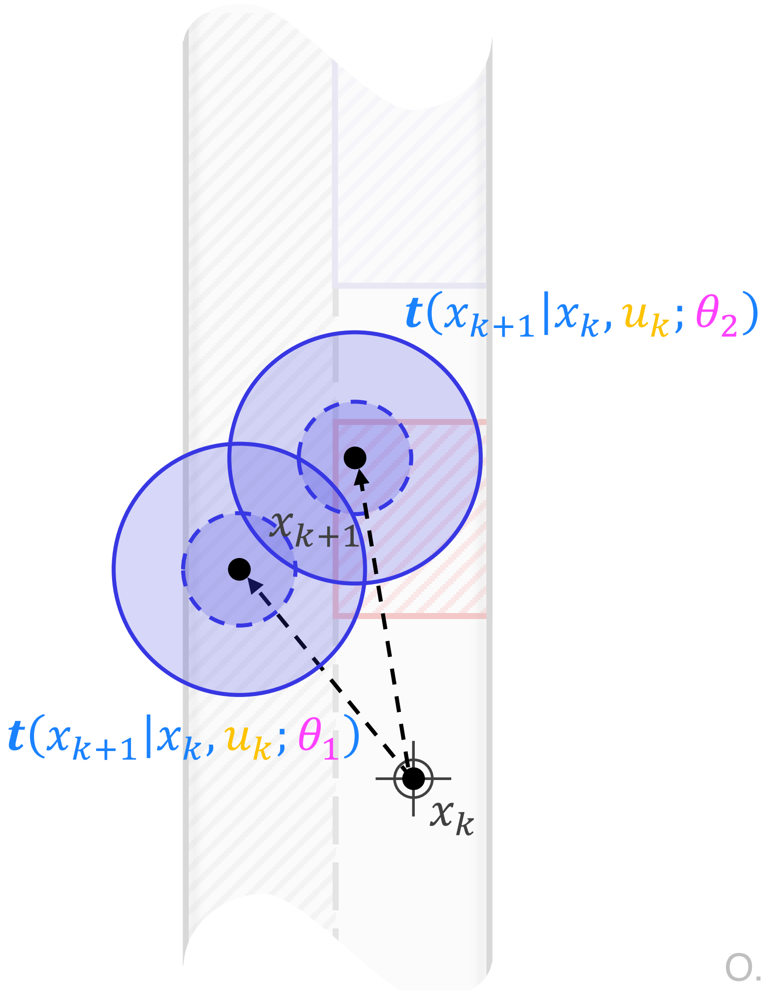
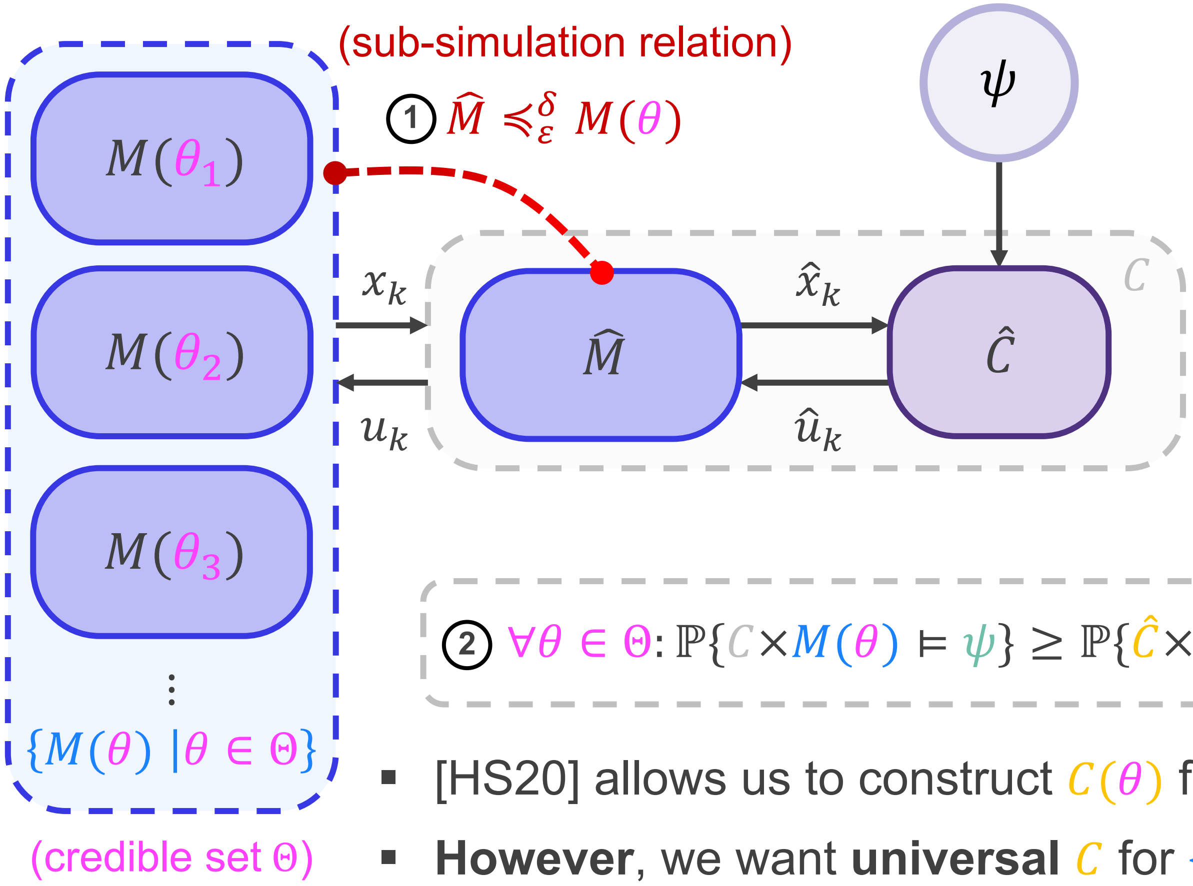
Correct-by-Design Control of Uncertain Systems

Sub-Simulation Relations (SSR)





② $\forall \theta \in \Theta: \mathbb{P}\{C \times M(\theta) \vDash \psi\} \geq \mathbb{P}\{\hat{C} \times \hat{M} \vDash \psi\} \geq p_\psi$

- [HS20] allows us to construct $C(\theta)$ for $M(\theta)$
- **However**, we want **universal** C for $\{M(\theta)\ |\theta \in \Theta\}$

O. Schön et al. (2022) *Correct-by-design control of parametric stochastic systems. CDC'22.*

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Correct-by-Design Control of Uncertain Systems

Layered solution

Procedure:

1) Three layers of models: $\mathrm{M}(\theta)$, $\widehat{\mathrm{M}}$, $\widetilde{\mathrm{M}}$
2) Two simulation relations:

$\widehat{\mathrm{M}} \preccurlyeq_{\varepsilon_1}^{\delta_1} \mathrm{M}(\theta)$ [O. Schön et al. (2022)]

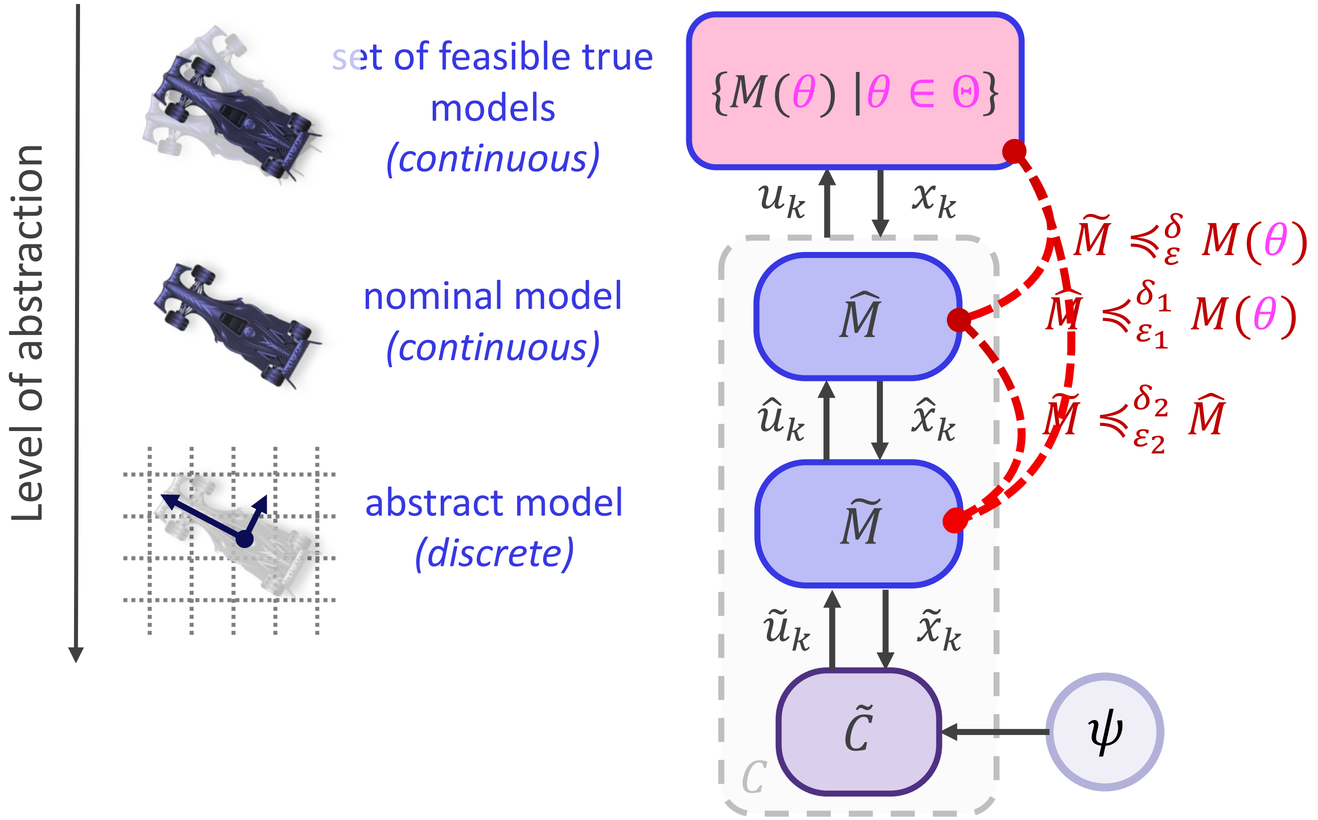
$\widetilde{\mathrm{M}} \preccurlyeq_{\varepsilon_2}^{\delta_2} \widehat{M}$ [S. Haesaert et al. (2017)]

→ $\widetilde{\mathrm{M}} \preccurlyeq_{\varepsilon}^{\delta} \mathrm{M}(\theta),\ \delta = \delta_1 + \delta_2,\ \ \varepsilon = \varepsilon_1 + \varepsilon_2$

3) Robust controller $\tilde{C}$ for $\widetilde{M}$

[S. Haesaert & S. Soudjani (2020)]

✓Robust satisfaction probability for C on $\{M(\theta)\ |\theta \in \Theta\}$



O. Schön et al. (2022) *Correct-by-design control of parametric stochastic systems. CDC'22.*

S. Haesaert & S. Soudjani (2020) *Robust dynamic programming for temporal logic control of stochastic systems. TAC.*

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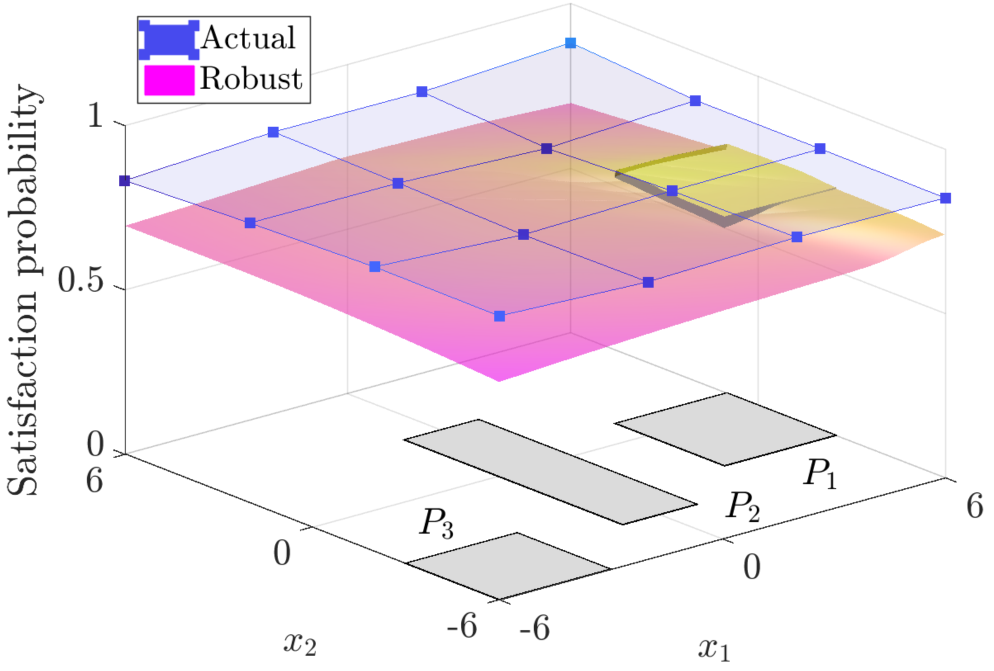
Numerical Results

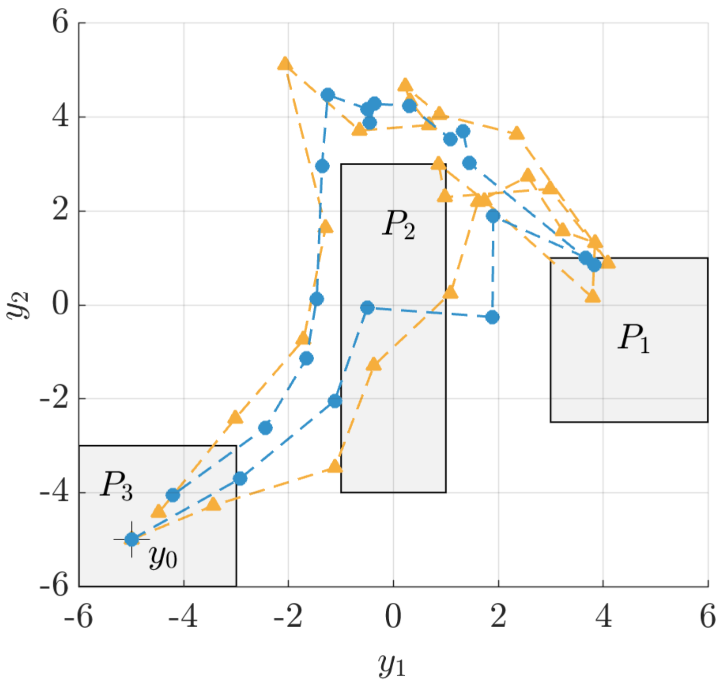
Linear and nonlinear case studies

Applicable to linear and nonlinear system with Gaussian disturbances
Varying amounts of data
Lower bound on satisfaction probability



$$\psi = \Diamond (P_1 \wedge (\neg P_2 \ \mathsf{U} \ P_3))$$

$$M: \begin{cases} x_{k+1} = \theta^T [x_k; u_k] + w_k \\ y_k = x_k. \end{cases} \quad w_k \sim p_w(\cdot)$$



Computed with SySCoRe
https://github.com/BirgitVanHuijgevoort/SySCoRe-software

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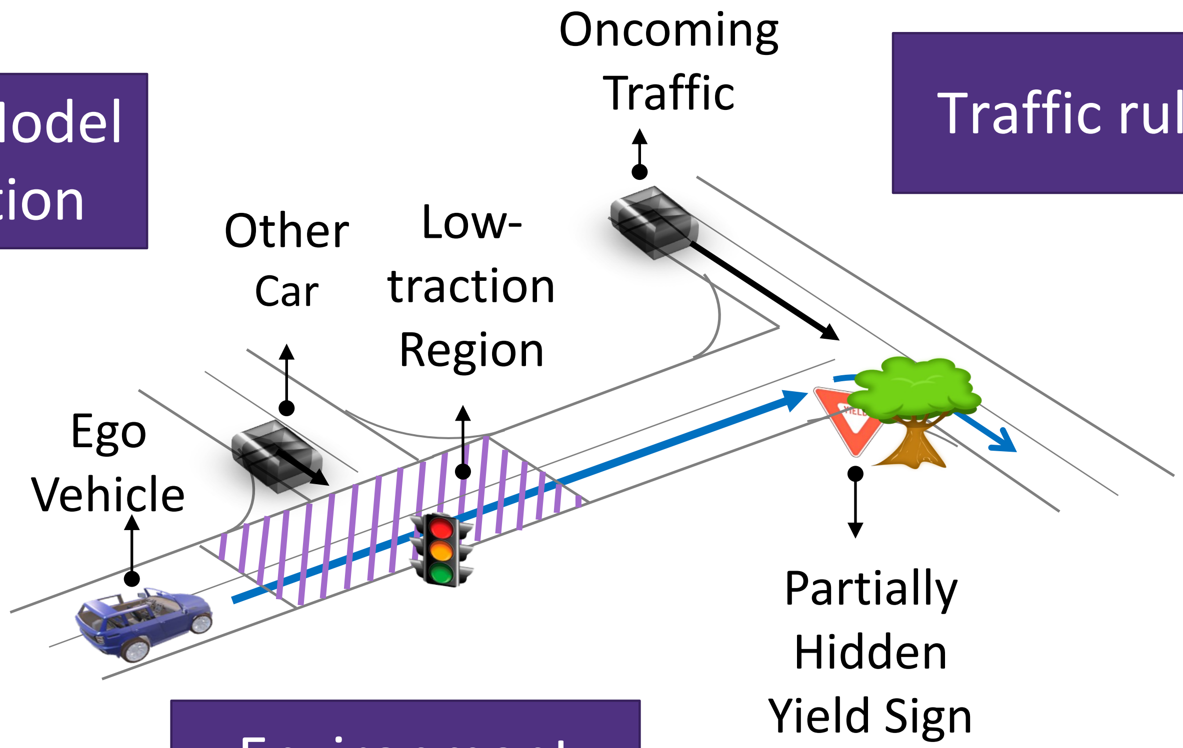
Dealing with perception data and environment uncertainty

Motion Model Abstraction

Traffic rules



Environment knowledge

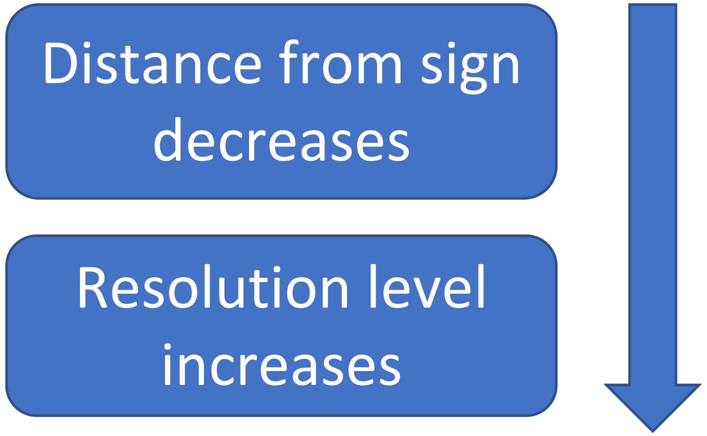
Kamale, Disha, Sofie Haesaert, and Cristian-Ioan Vasile. "Cautious Planning with Incremental Symbolic Perception: Designing Verified Reactive Driving Maneuvers." ICRA 2023

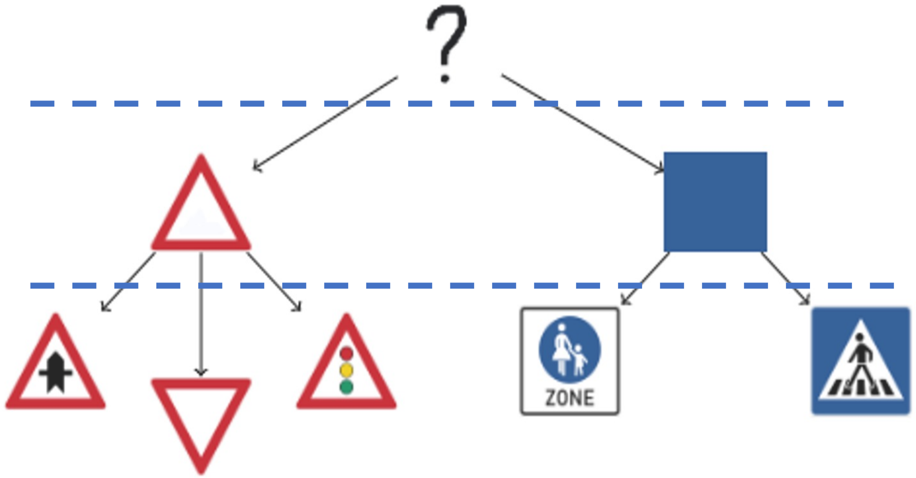
Incremental perception of environment

Uncertainty reduces while you get closer

Perception

Capture perception uncertainty dynamics as a symbolic finite state model





Kamale, Disha, Sofie Haesaert, and Cristian-Ioan Vasile. "Cautious Planning with Incremental Symbolic Perception: Designing Verified Reactive Driving Maneuvers." ICRA 2023

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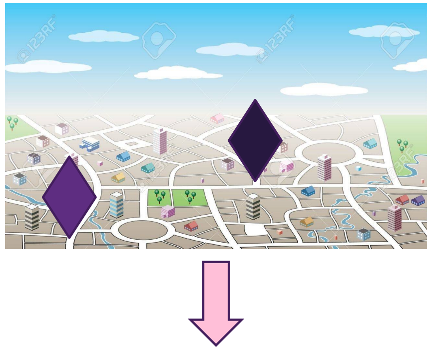
Dealing with incremental perception of environment

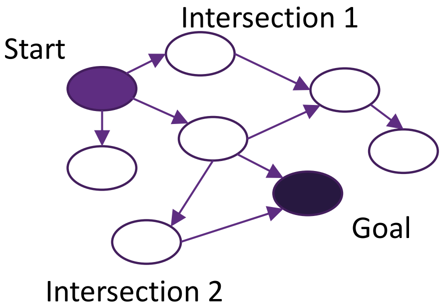
Traffic Rules

Traffic and safety rules as
Temporal logic formula

Encoded as GR(1) (Assume Guarantee) and solved with Omega / TuLiP

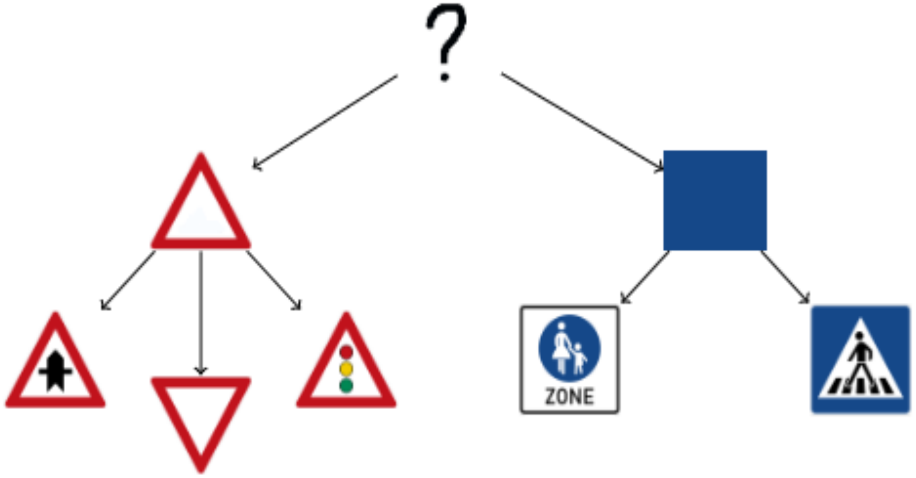
Motion Model Abstraction





Perception

New finite symbolic models for incremental perception of the environment



Kamale, Disha, Sofie Haesaert, and Cristian-Ioan Vasile. "Cautious Planning with Incremental Symbolic Perception: Designing Verified Reactive Driving Maneuvers." ICRA 2023

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Approach

System Variables
Y = {stop, move, move_slowly, …}

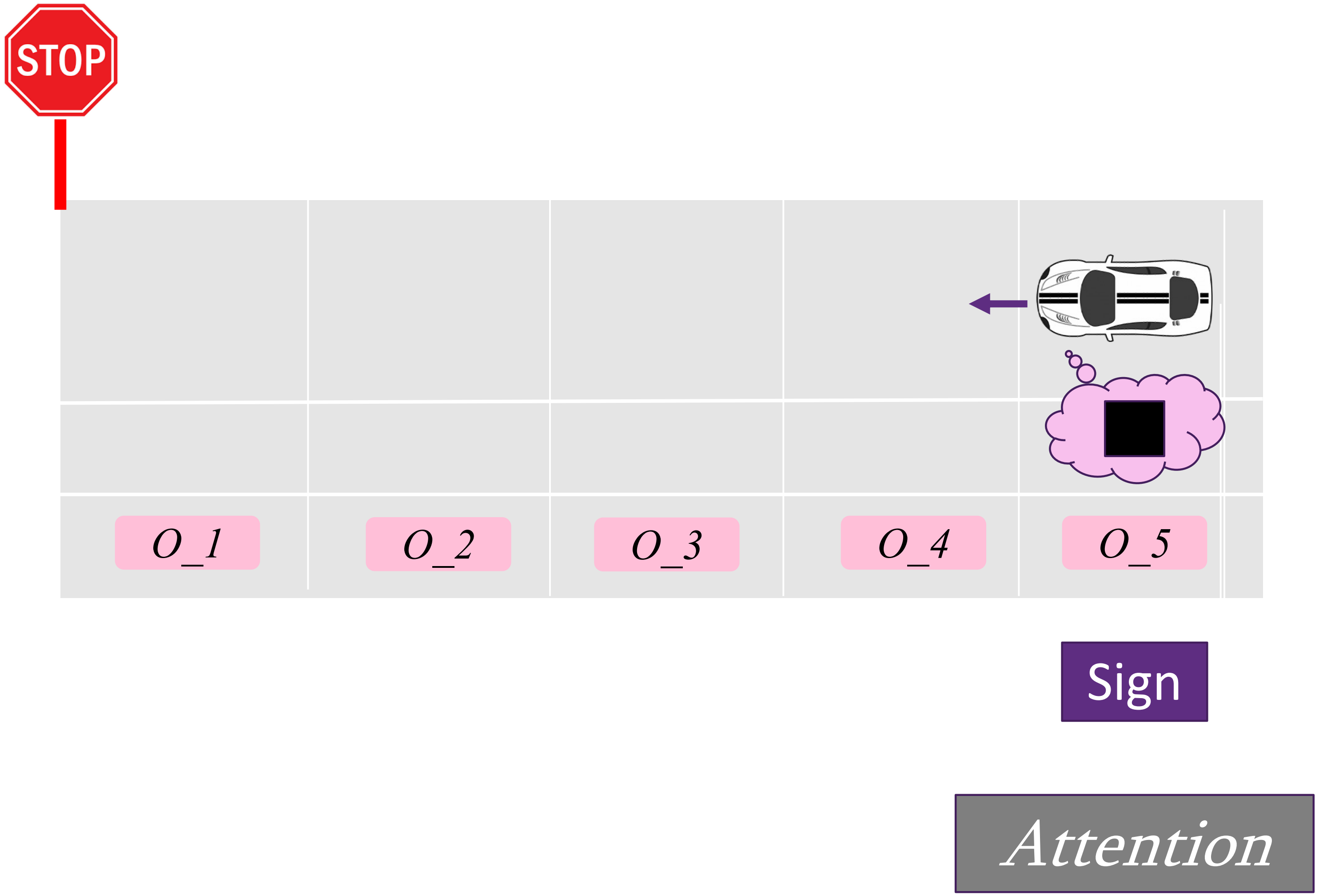
Symbolic Perception Variables
X = {stop_sign, octagonal, warning, …}

Safety | Initial States | Progress Conditions

2 player GR(1) game ->
Winning Control Plan

Wongpiromsarn, T., et.al. TuLiP: a software toolbox for receding horizon temporal logic planning. HSCC'11

Simulation



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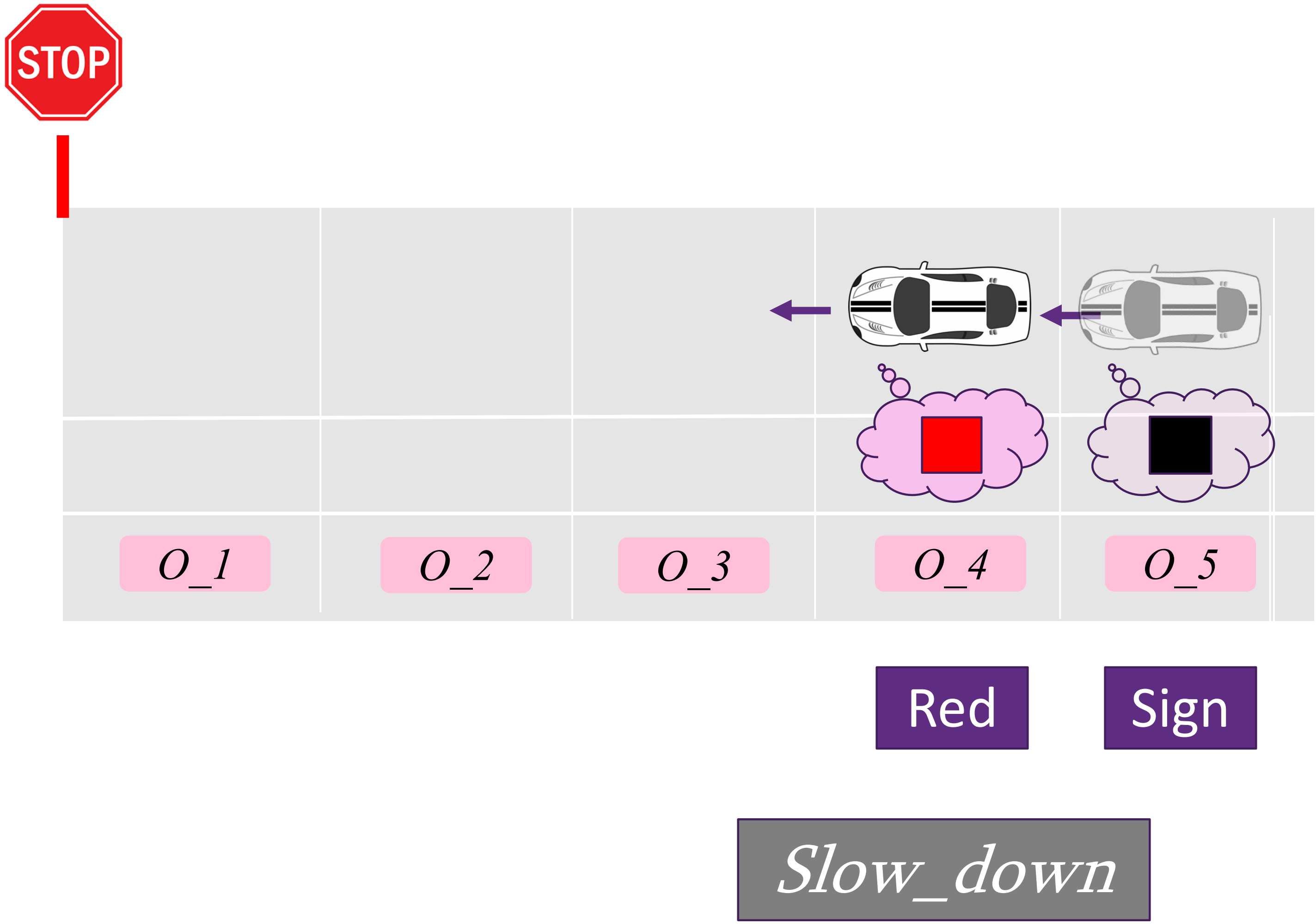
Simulation



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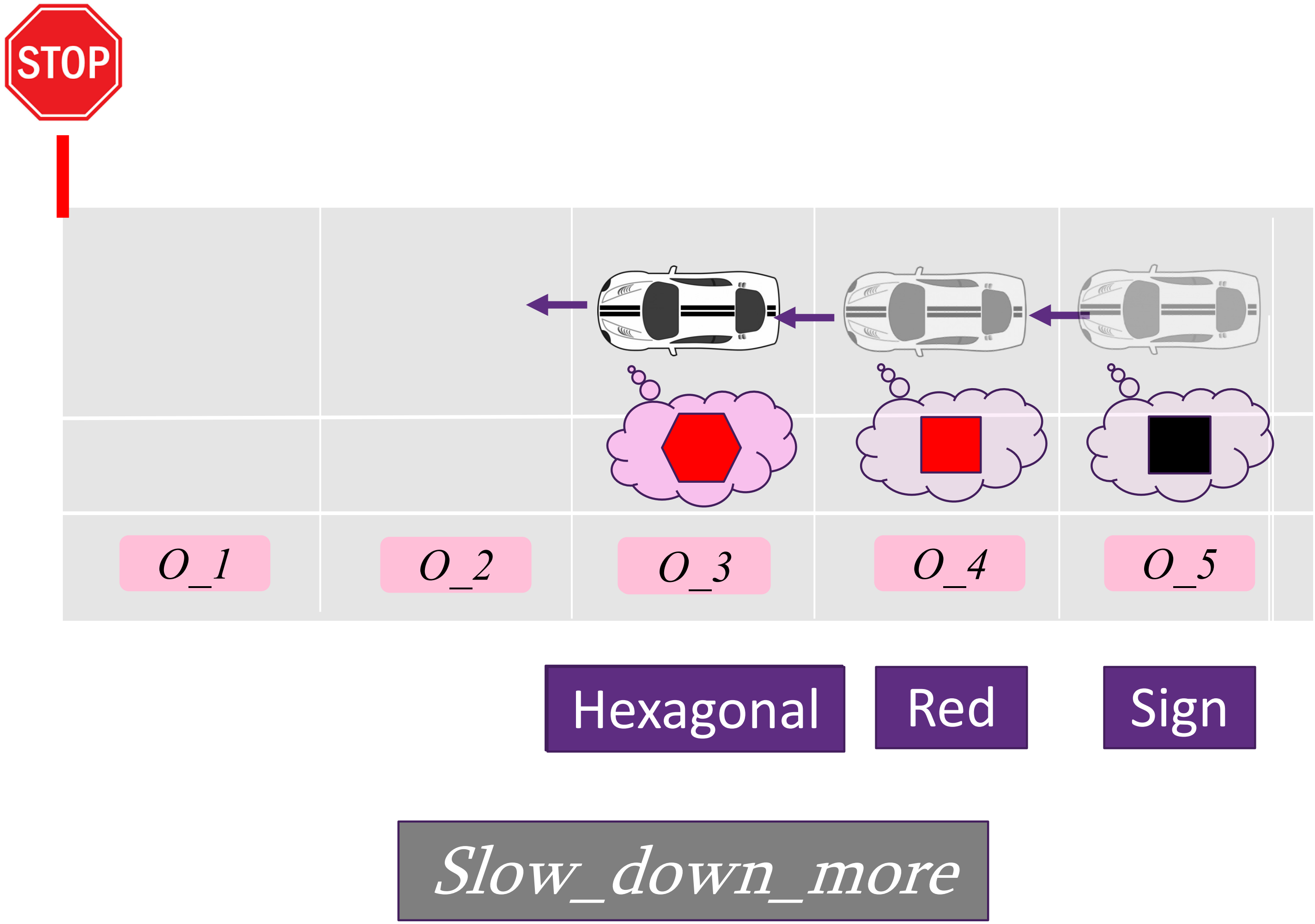
Simulation



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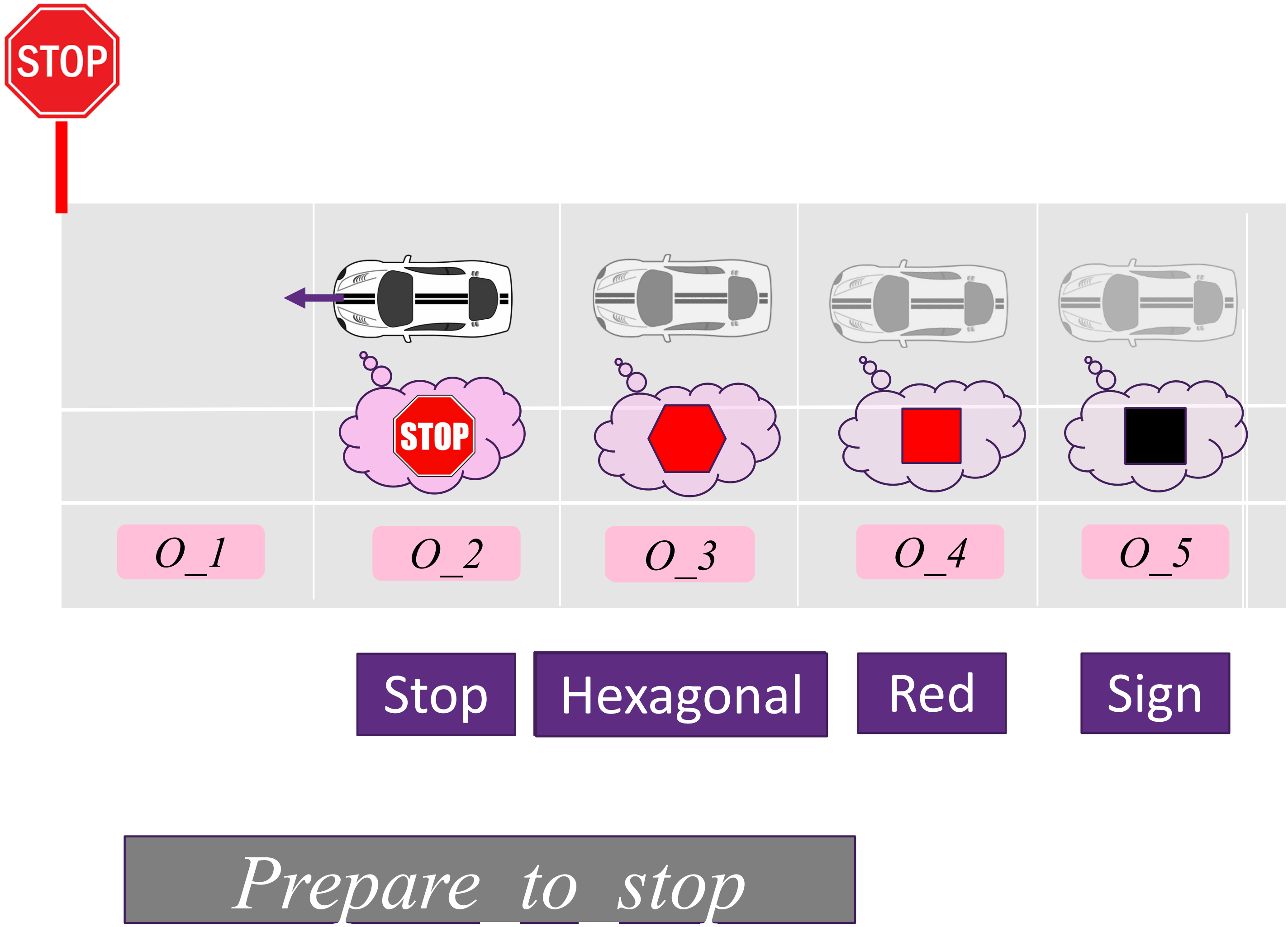

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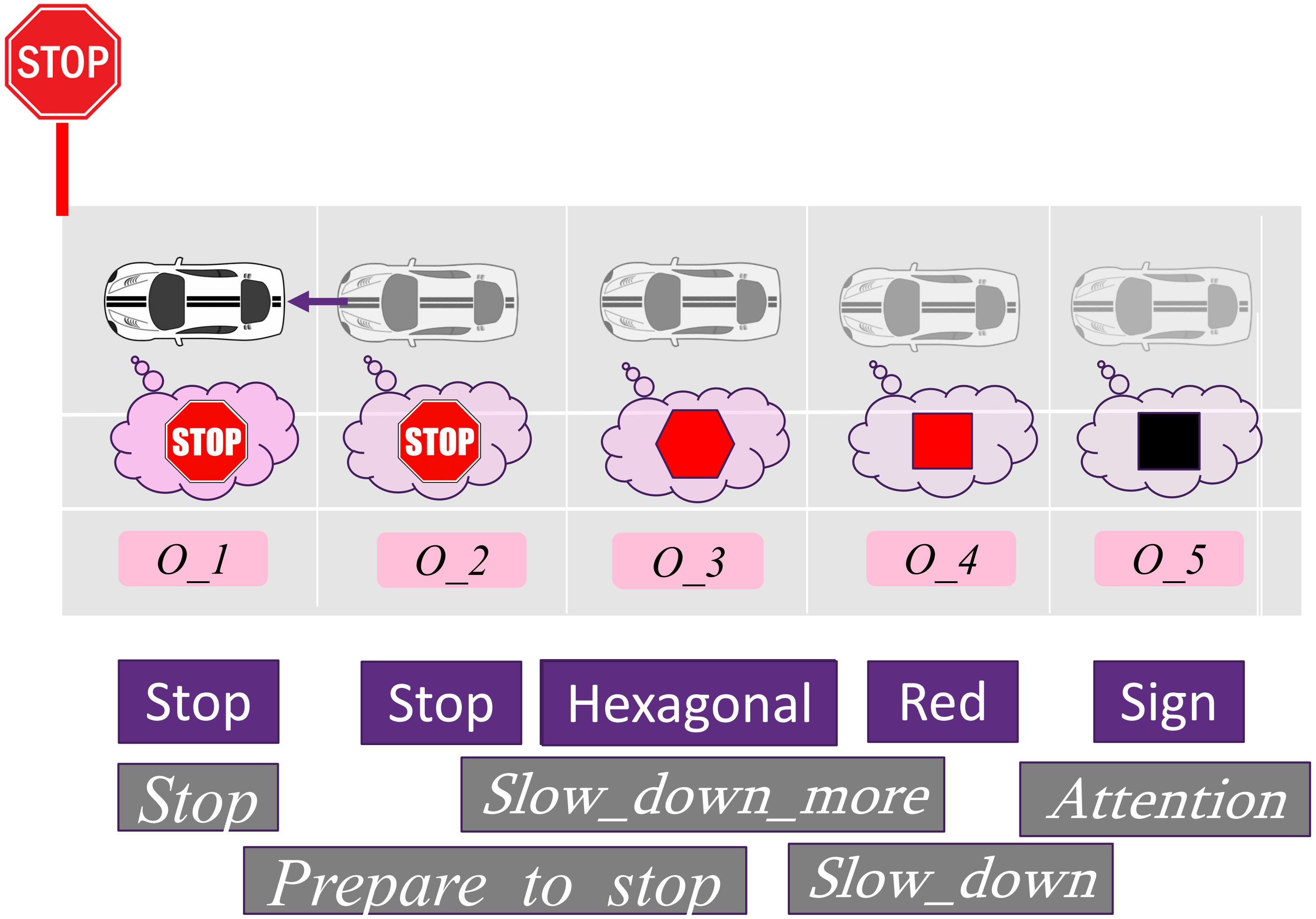

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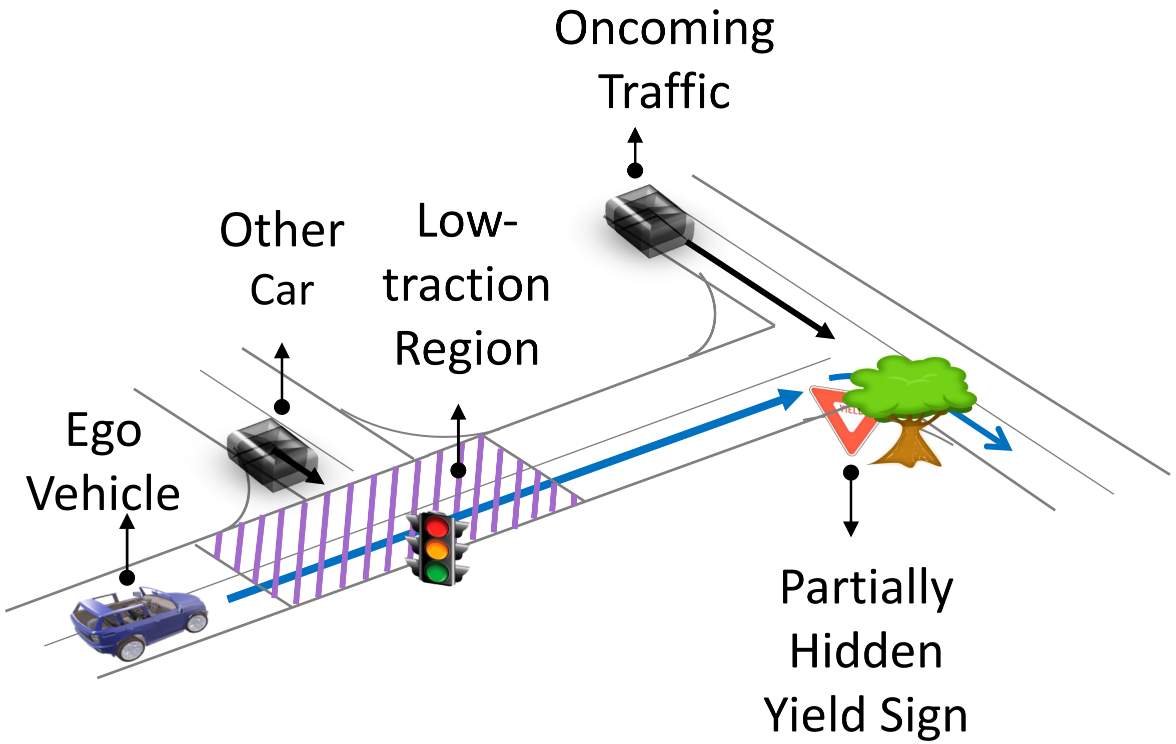
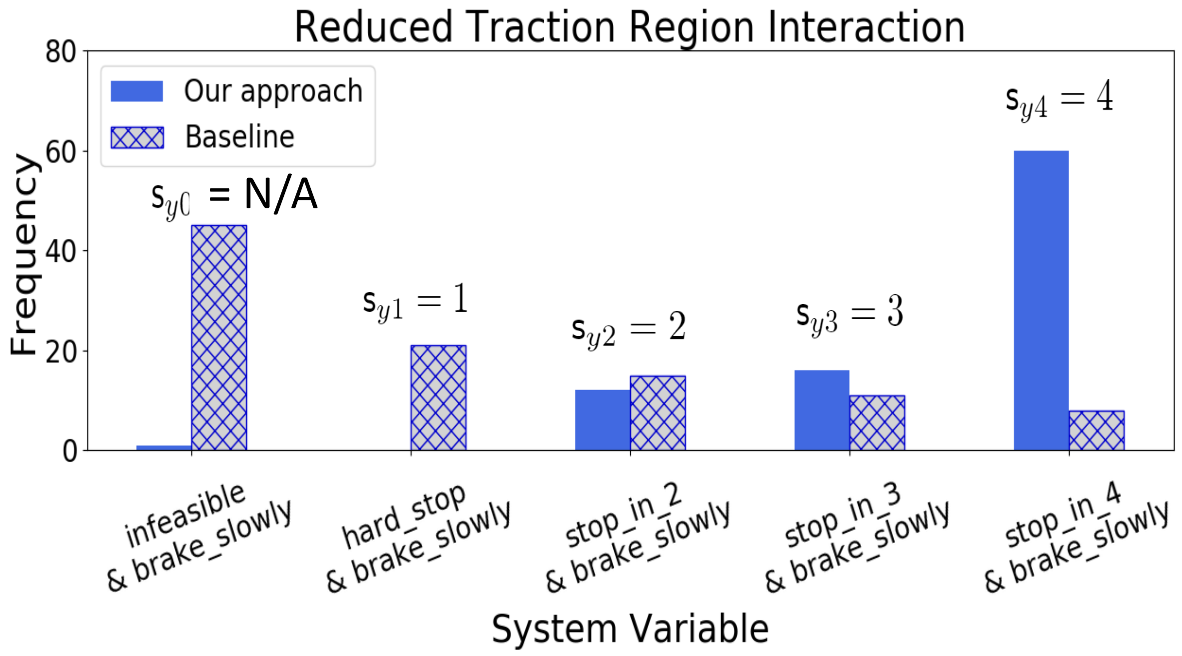
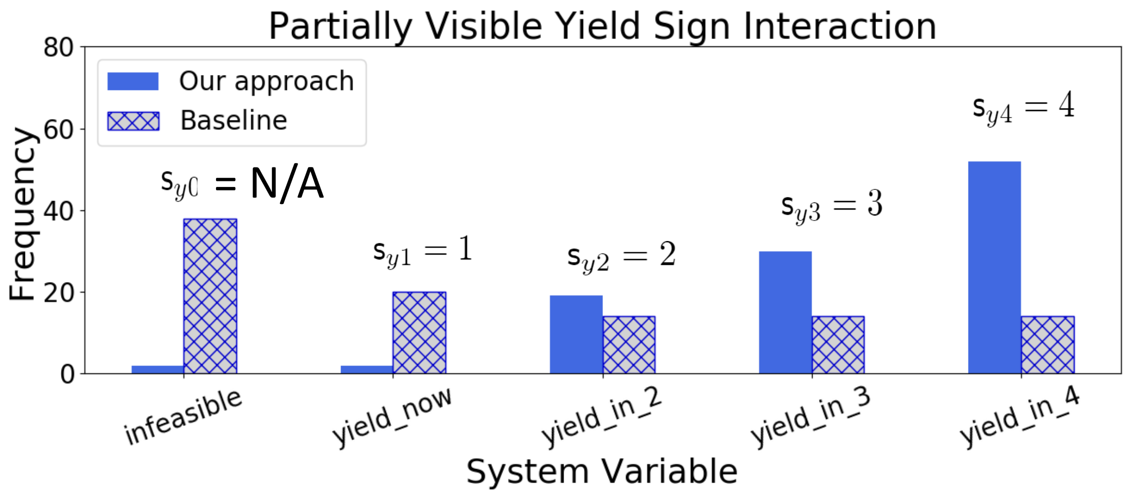
Simulation



Oncoming Traffic
Other Car
Low-traction Region
Ego Vehicle
Partially Hidden Yield Sign



Reduced Traction Region Interaction
Our approach
Baseline
s_{y0} = N/A
$s_{y1} = 1$
$s_{y2} = 2$
$s_{y3} = 3$
$s_{y4} = 4$
Frequency
0
20
40
60
80
infeasible & brake_slowly
hard_stop & brake_slowly
stop_in_2 & brake_slowly
stop_in_3 & brake_slowly
stop_in_4 & brake_slowly
System Variable



Partially Visible Yield Sign Interaction
Our approach
Baseline
s_{y0} = N/A
$s_{y1} = 1$
$s_{y2} = 2$
$s_{y3} = 3$
$s_{y4} = 4$
Frequency
0
20
40
60
80
infeasible
yield_now
yield_in_2
yield_in_3
yield_in_4
System Variable

Future work

1. Dealing with partially observable stochastic systems with parametric uncertainty
2. Combining actively getting data and verified control into 1 computable online approach with temporal logic guarantees for stochastic systems
3. Risk-awareness based on perception data and environment uncertainty

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- Roland Toth (TU Eindhoven)

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