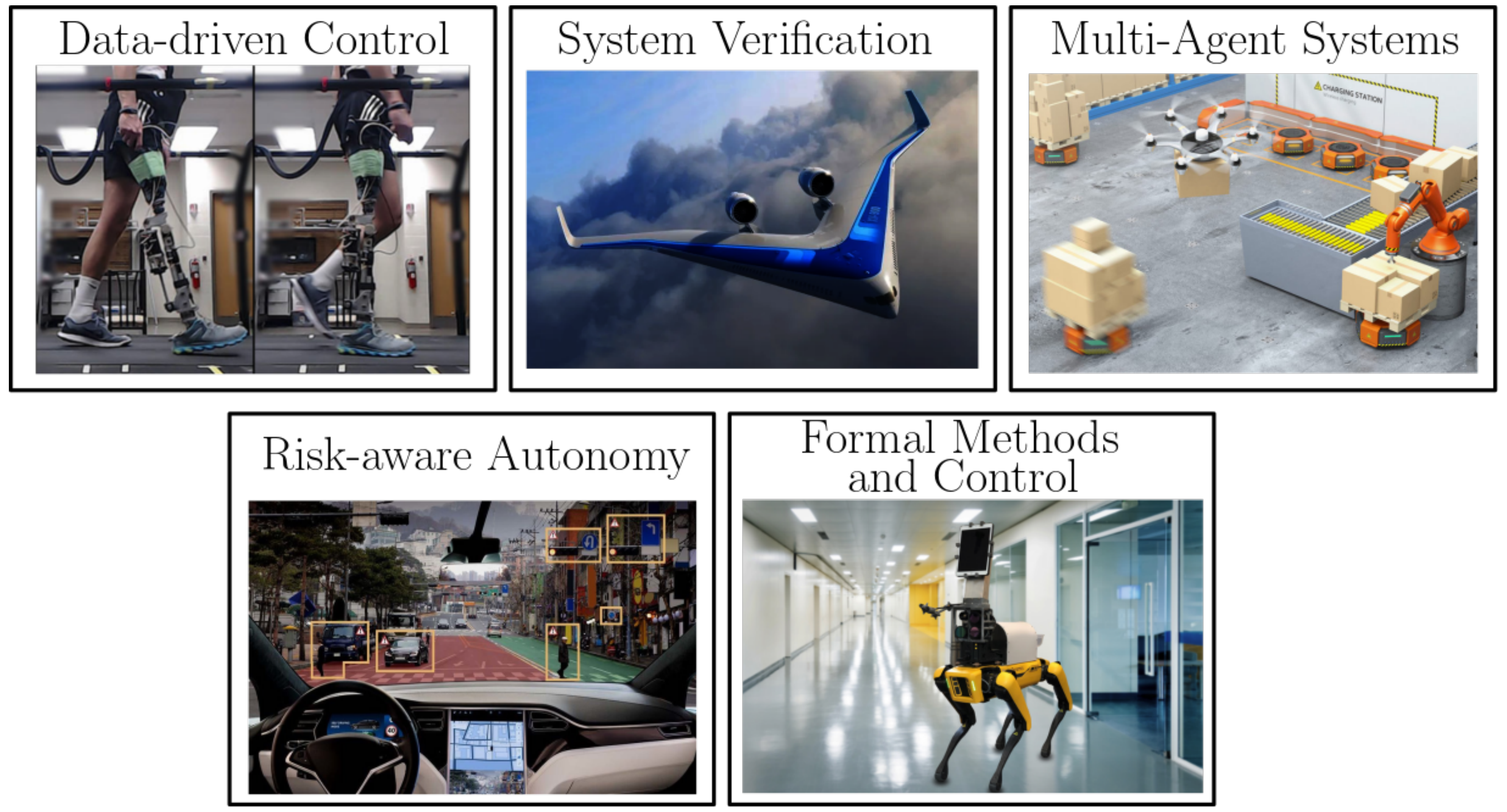
USC University of Southern California

Safe Control of Learning-Enabled Autonomous Systems using Conformal Prediction

Lars Lindemann

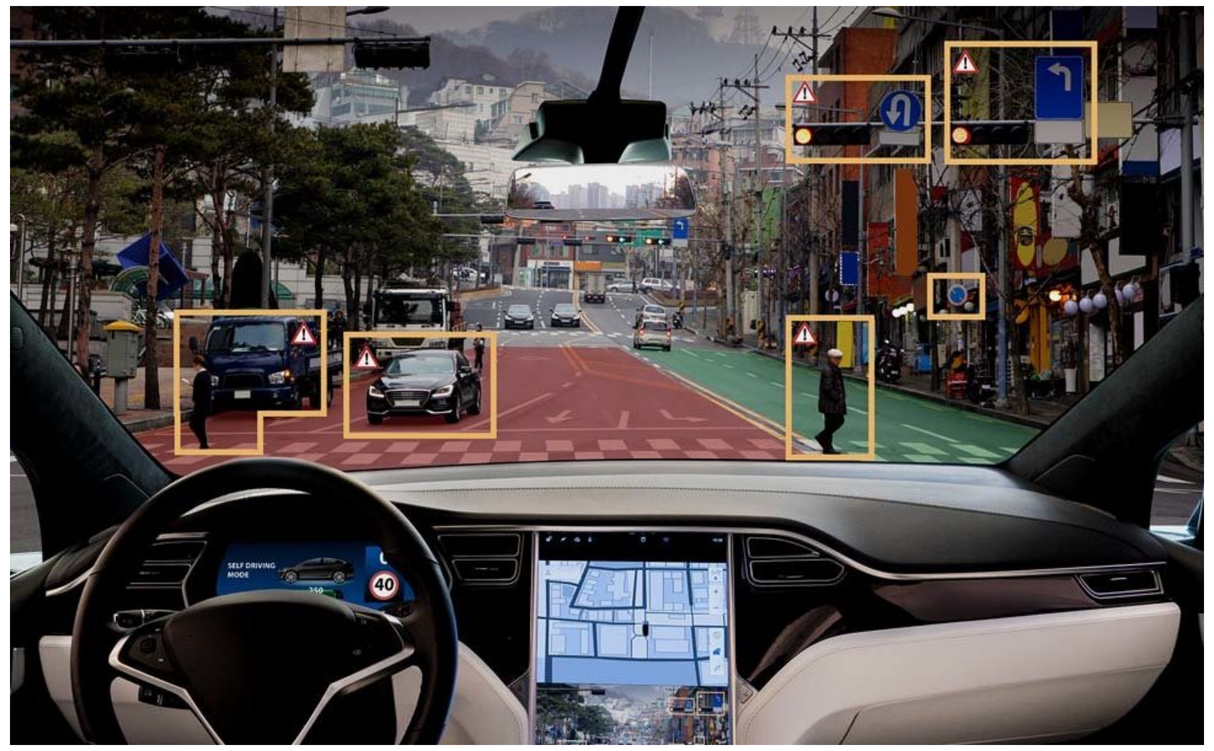
Safe Autonomy & Intelligent Distributed Systems (SAIDS) Lab



Our vision for autonomous systems

Control systems that operate autonomously in complex and open-ended environments.

Autonomous driving

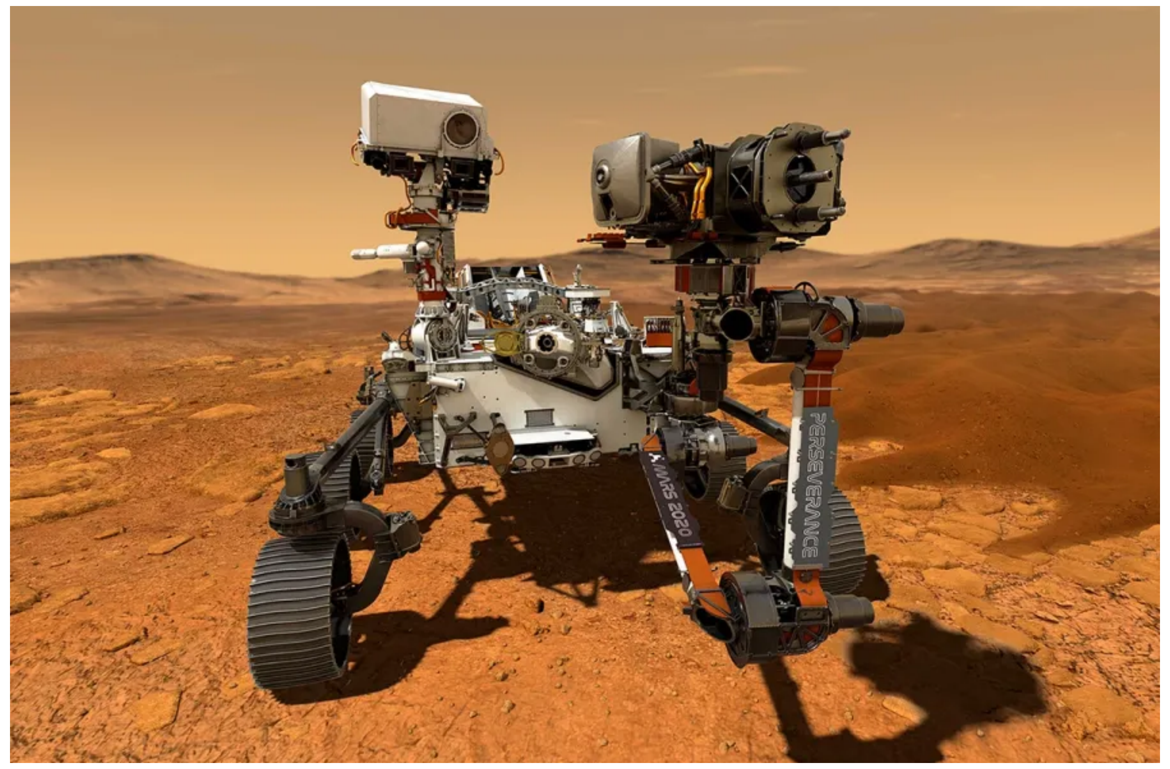


Robotics

Wildfire prevention

Warehouses

Aerospace

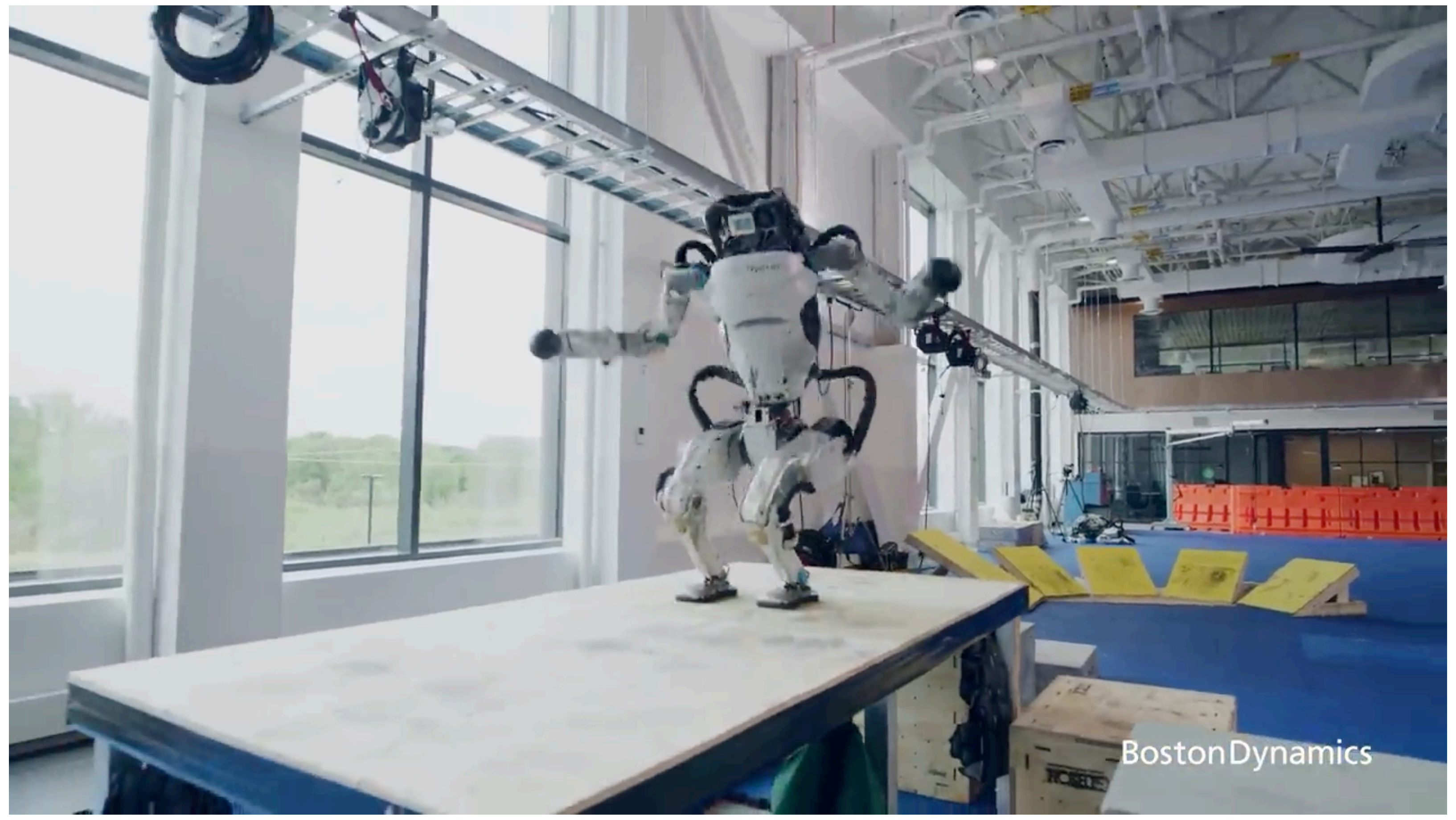


Agriculture

Smart Cities

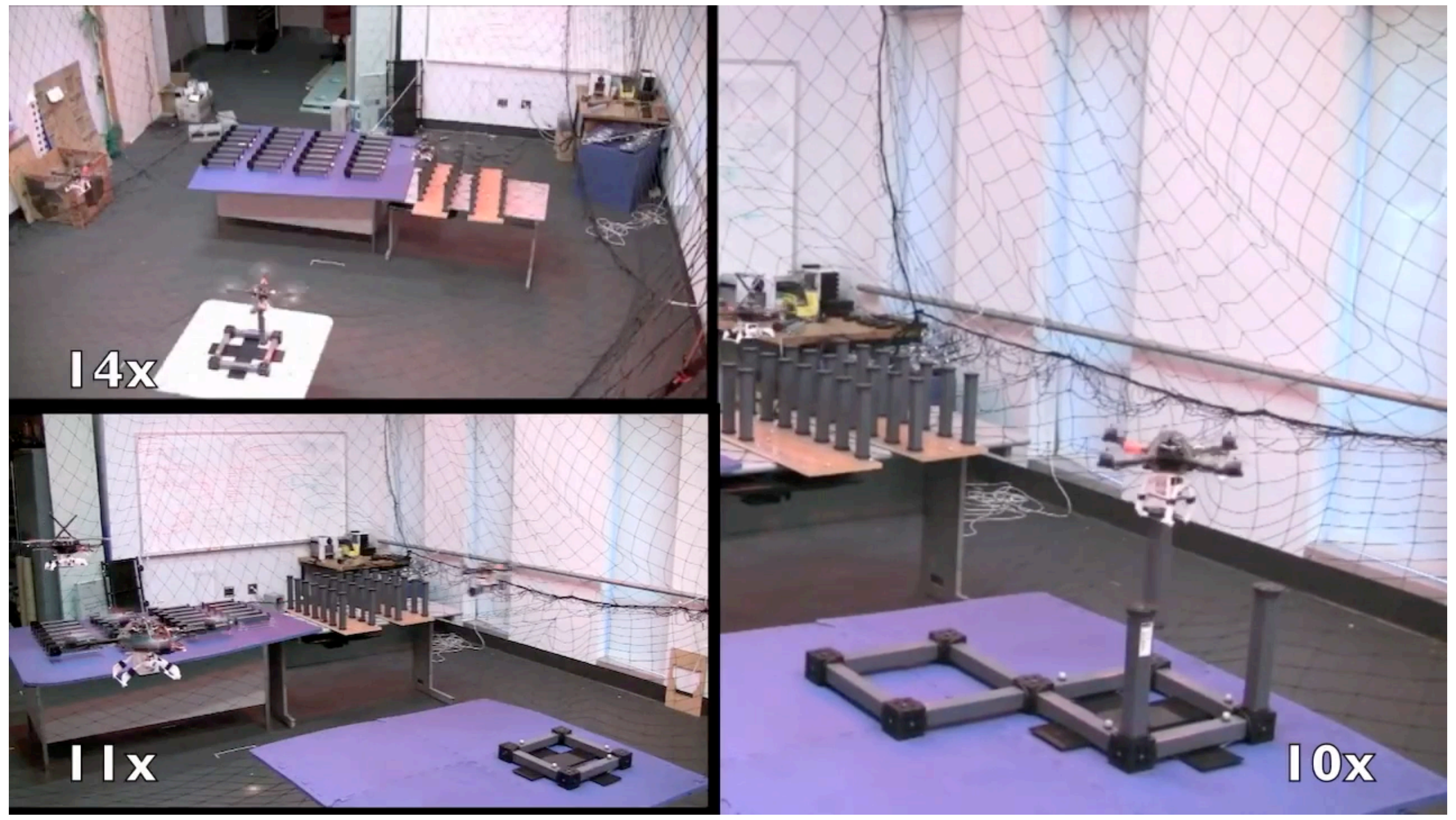
Health Care

Autonomous systems are a reality!



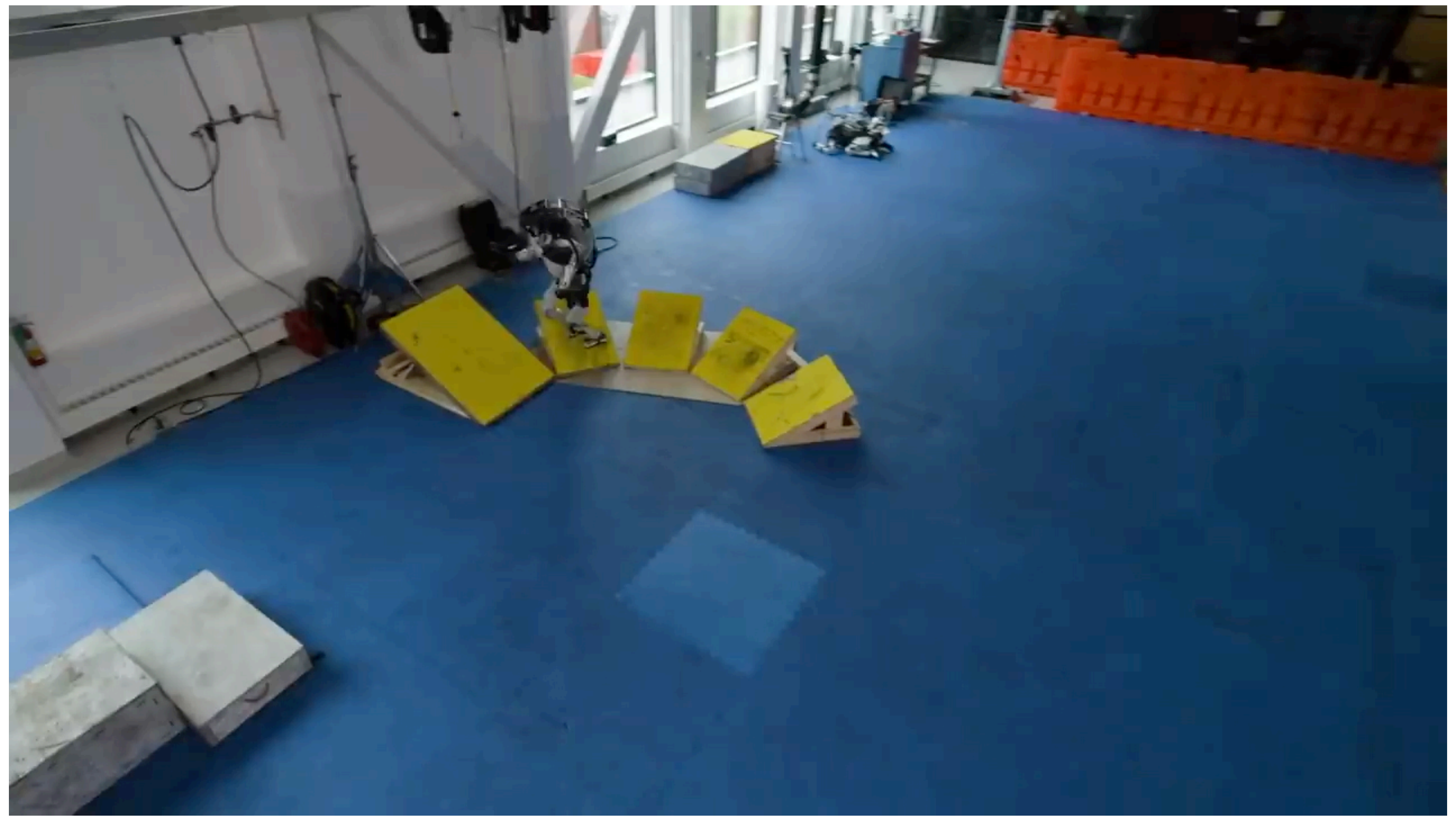
Boston Dynamics

Tesla



Kumar Lab (UPenn)

But are they safe?



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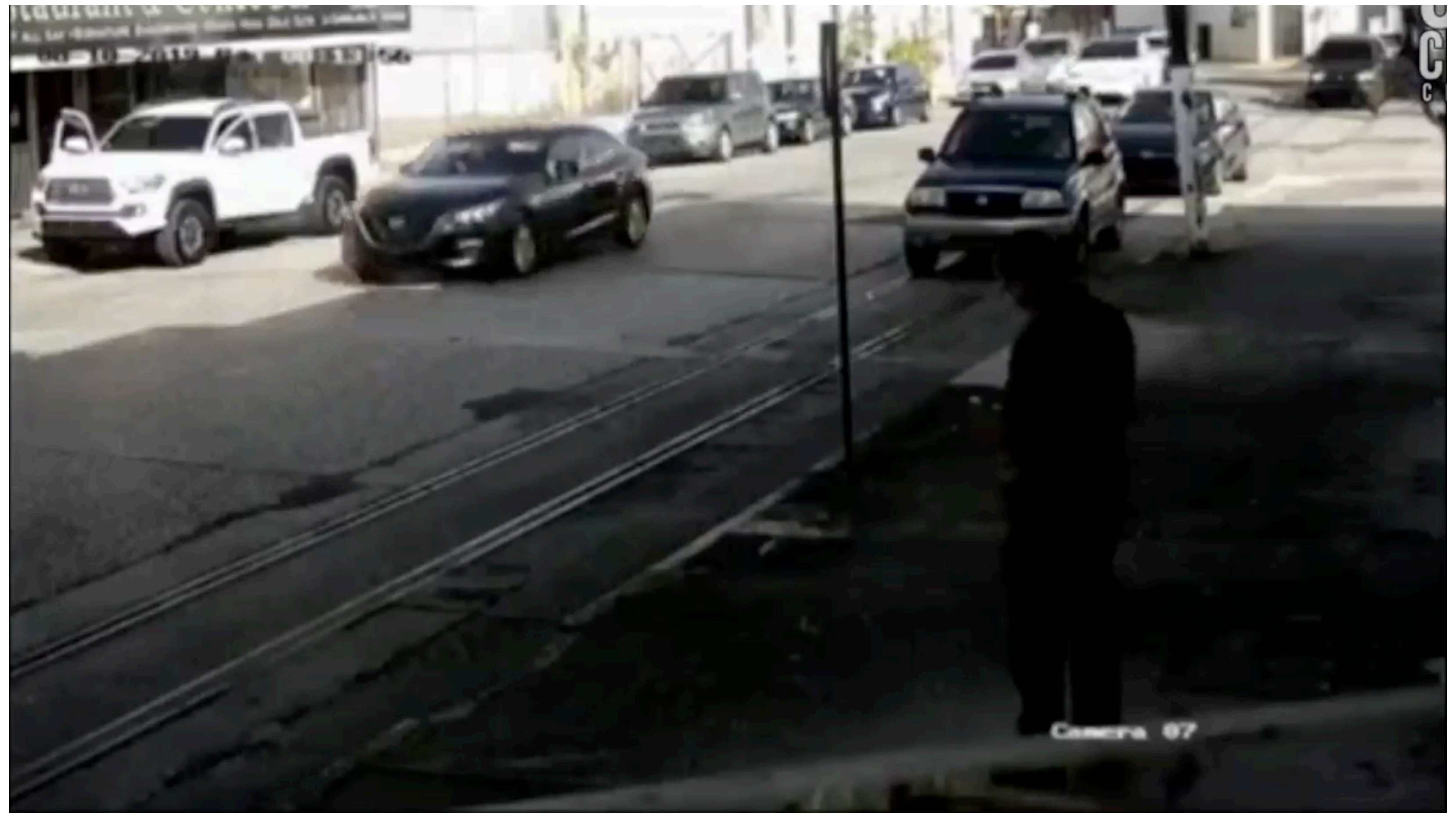

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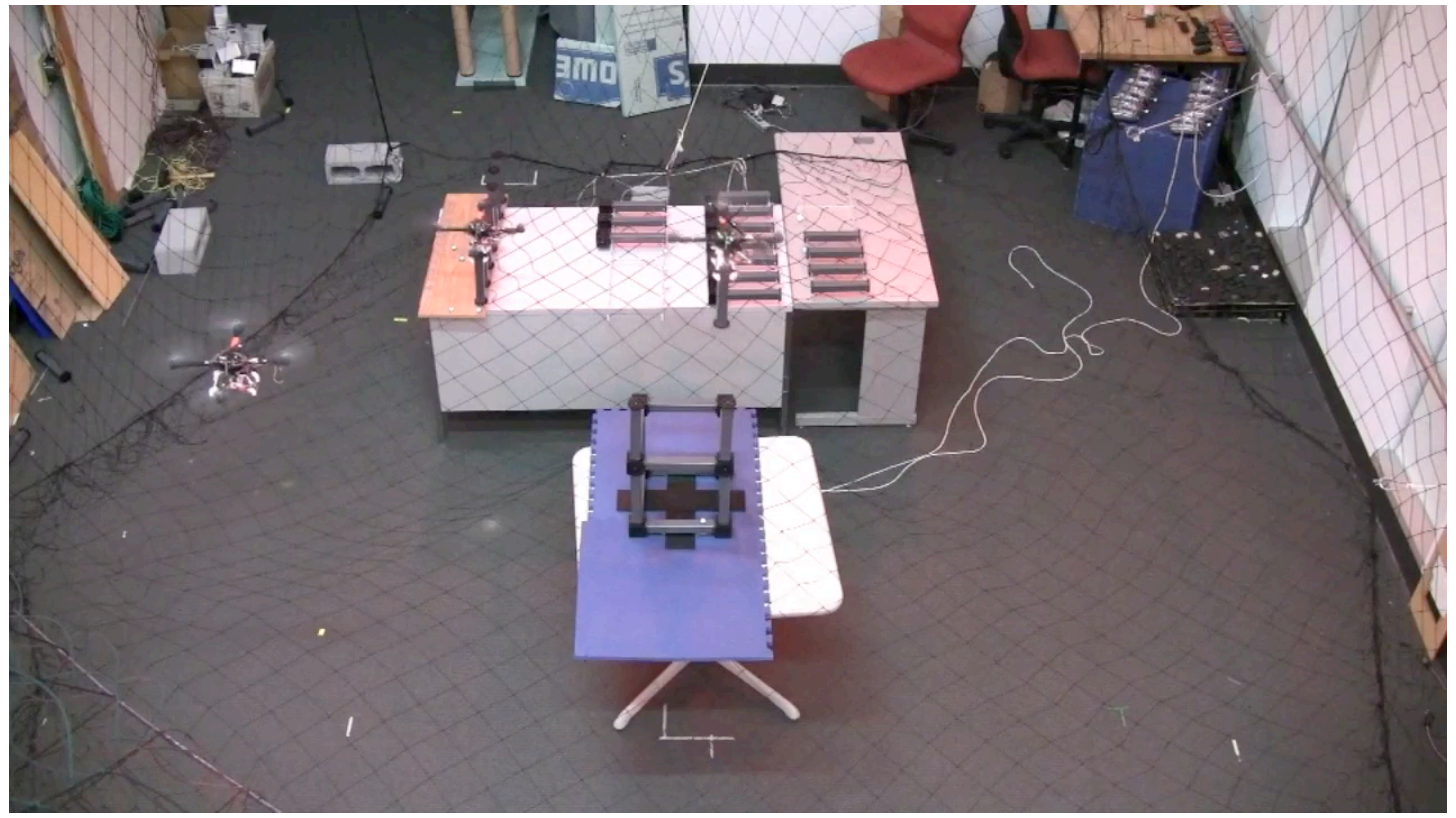
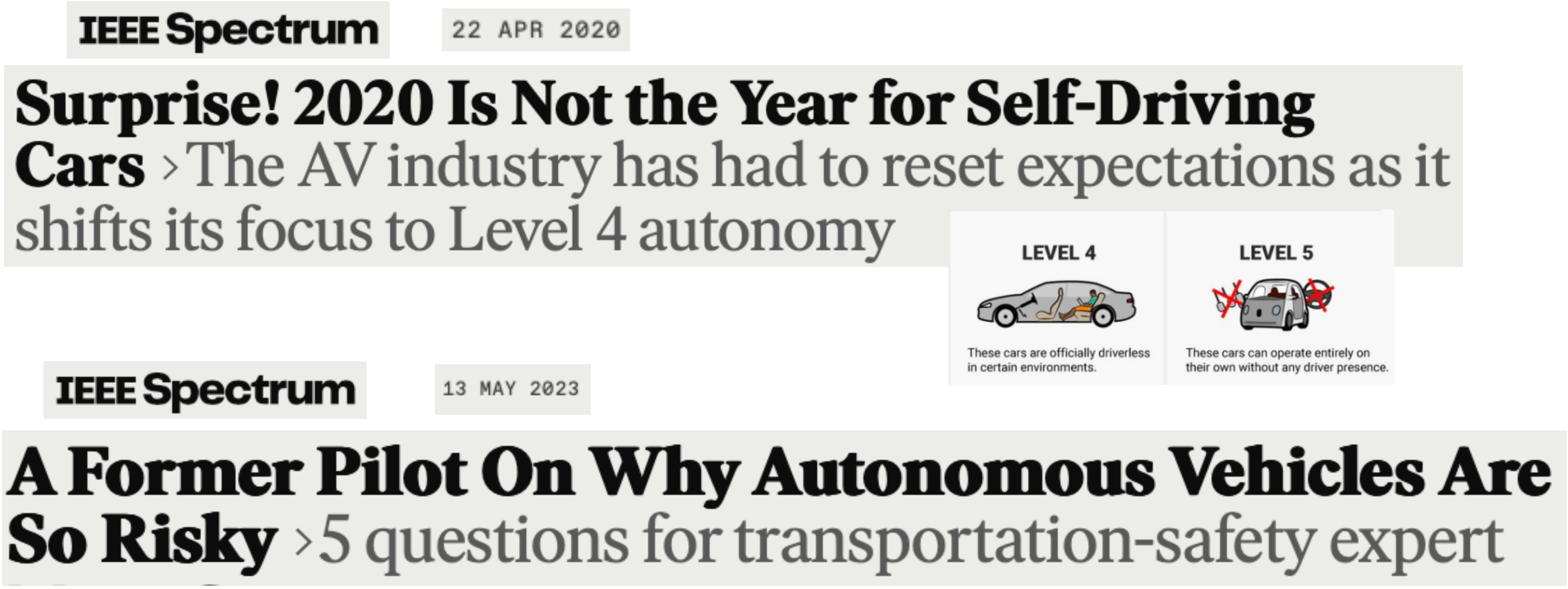


Kumar Lab (UPenn)

Learning-enabled autonomous systems are fragile



IEEE Spectrum 22 APR 2020

Surprise! 2020 Is Not the Year for Self-Driving Cars › The AV industry has had to reset expectations as it shifts its focus to Level 4 autonomy

IEEE Spectrum 13 MAY 2023

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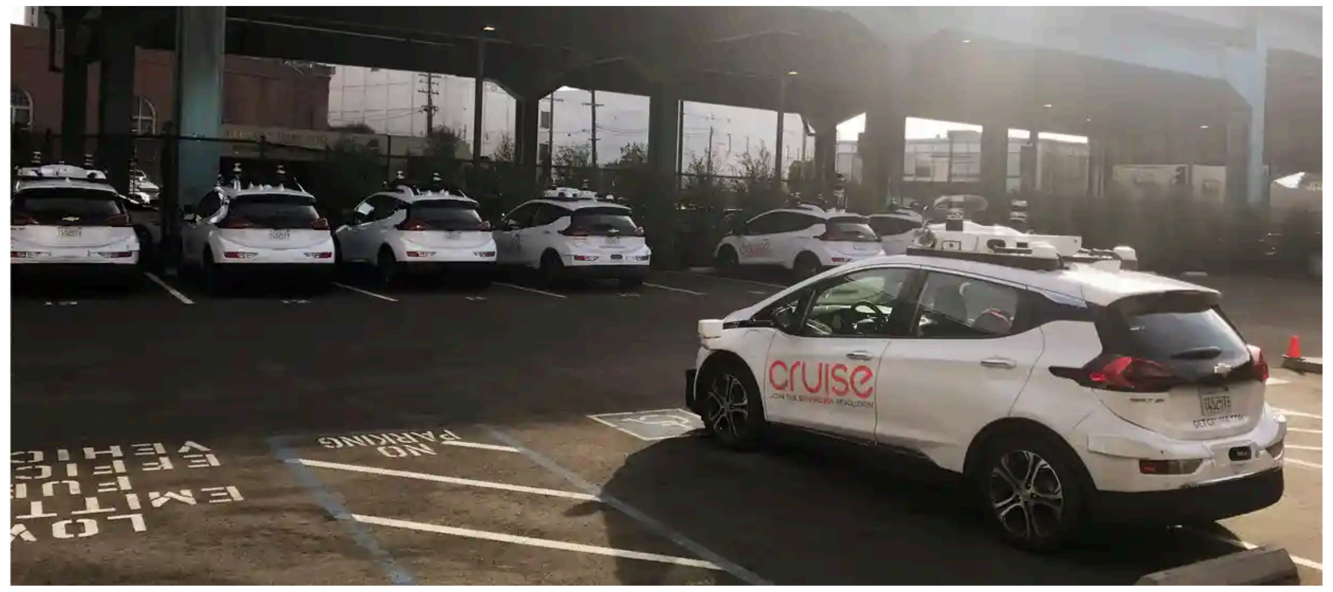
Cruise recalls all self-driving cars after grisly accident and California ban



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LEVEL 4

These cars are officially driverless in certain environments.

LEVEL 5

These cars can operate entirely on their own without any driver presence.

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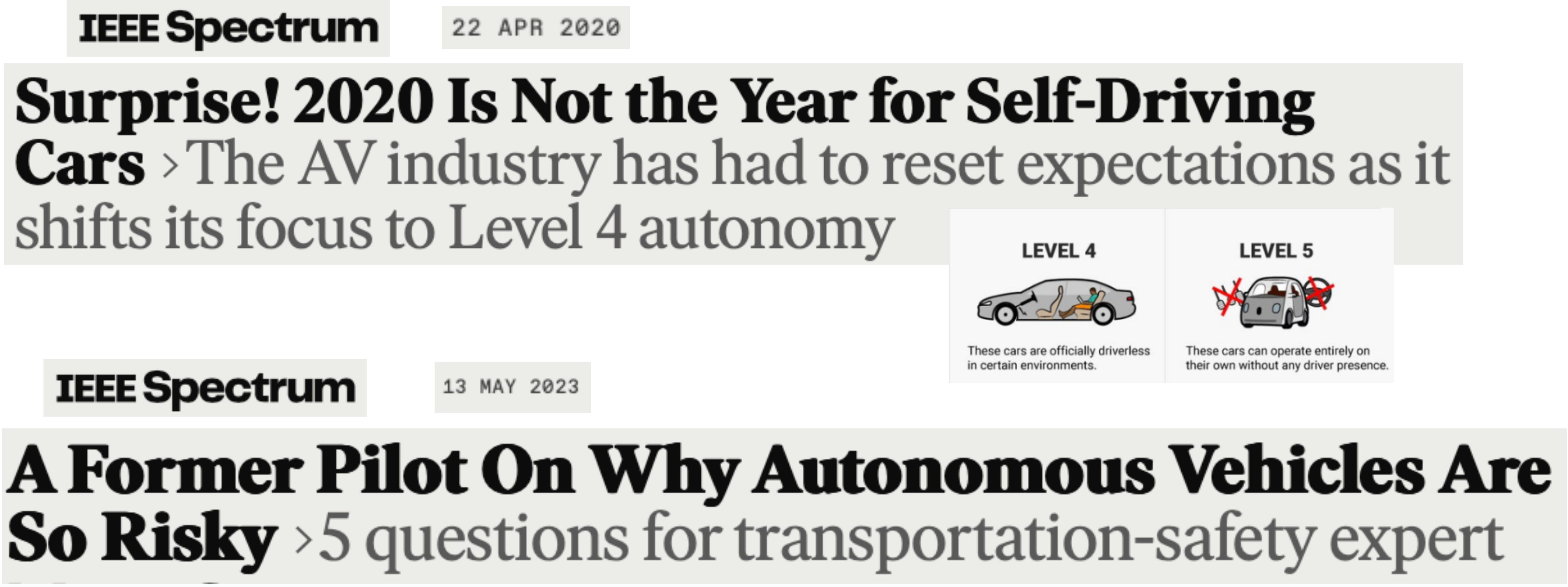
The way we train AI is fundamentally flawed

It's no secret that machine-learning models tuned and tweaked to near-perfect performance in the lab often fail in real settings.

Learning-enabled autonomous systems are fragile



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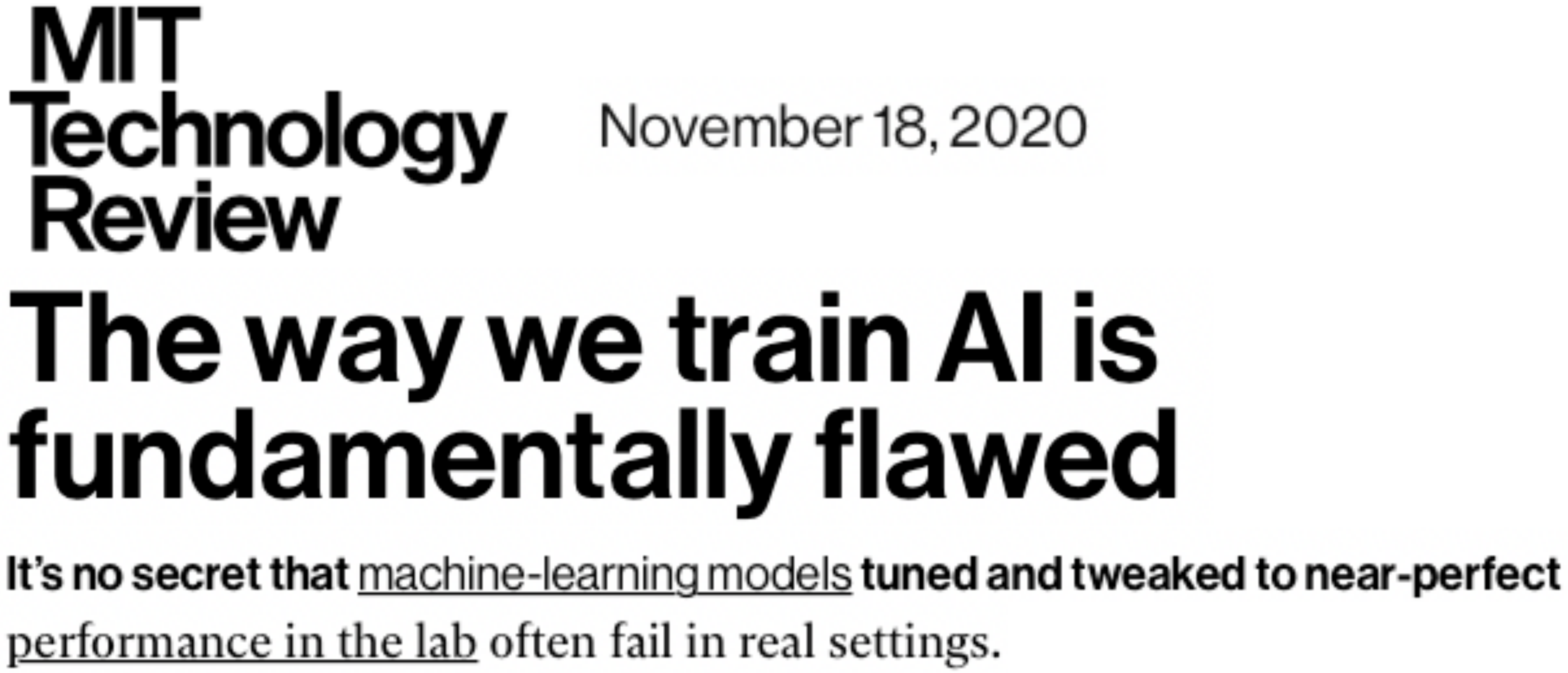

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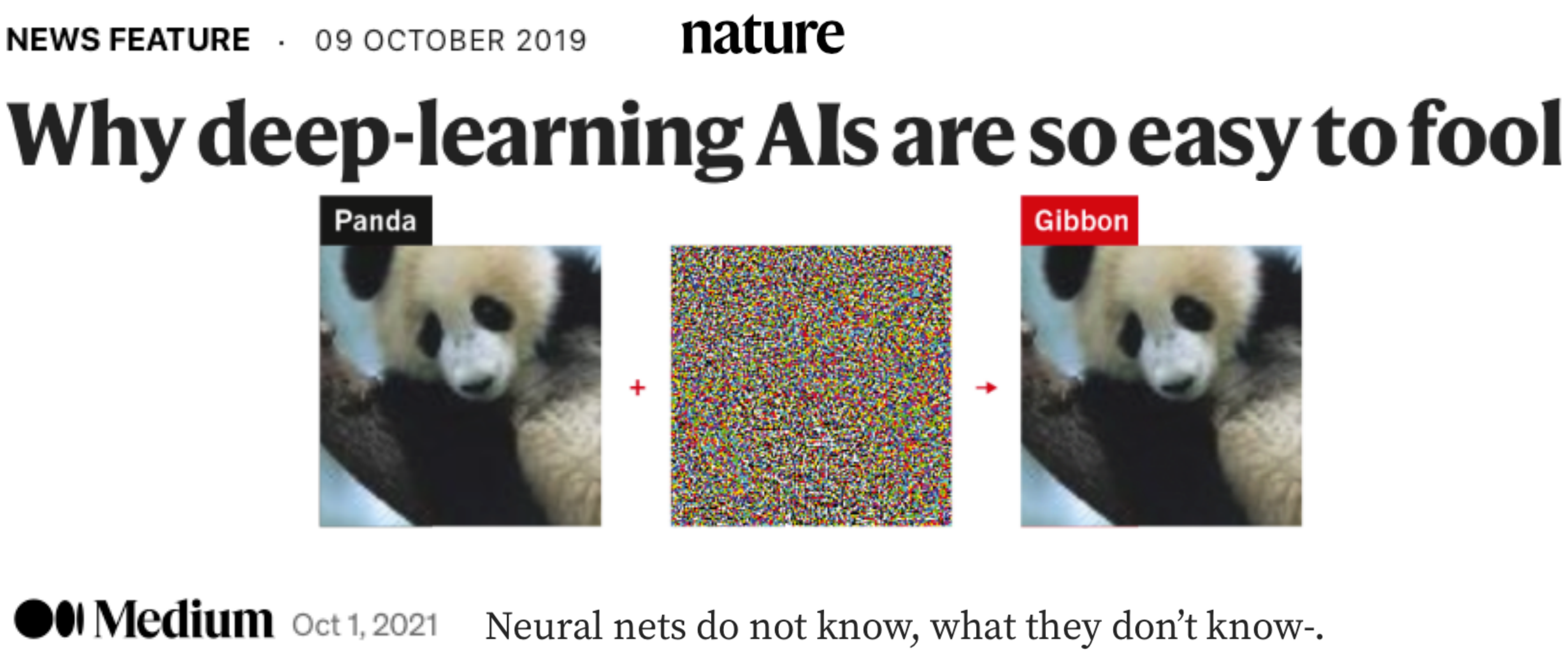
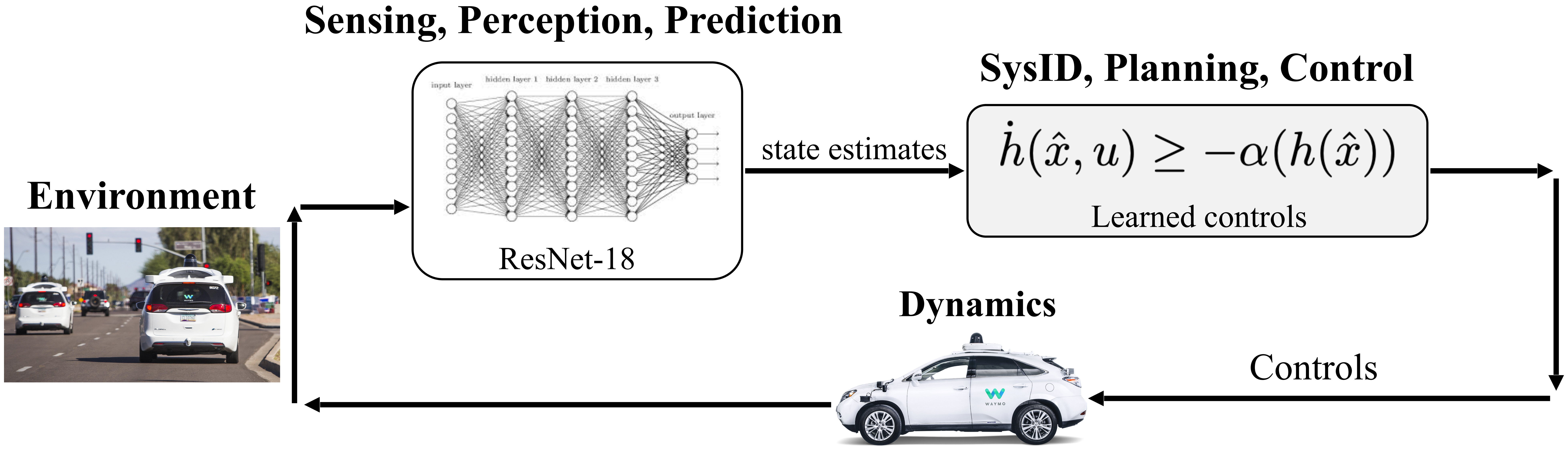


NEWS FEATURE · 09 OCTOBER 2019 **nature**

Why deep-learning AIs are so easy to fool

Medium Oct 1, 2021 Neural nets do not know, what they don't know-.

Safe control for learning-enabled autonomy



Environment

Sensing, Perception, Prediction

ResNet-18

state estimates

SysID, Planning, Control

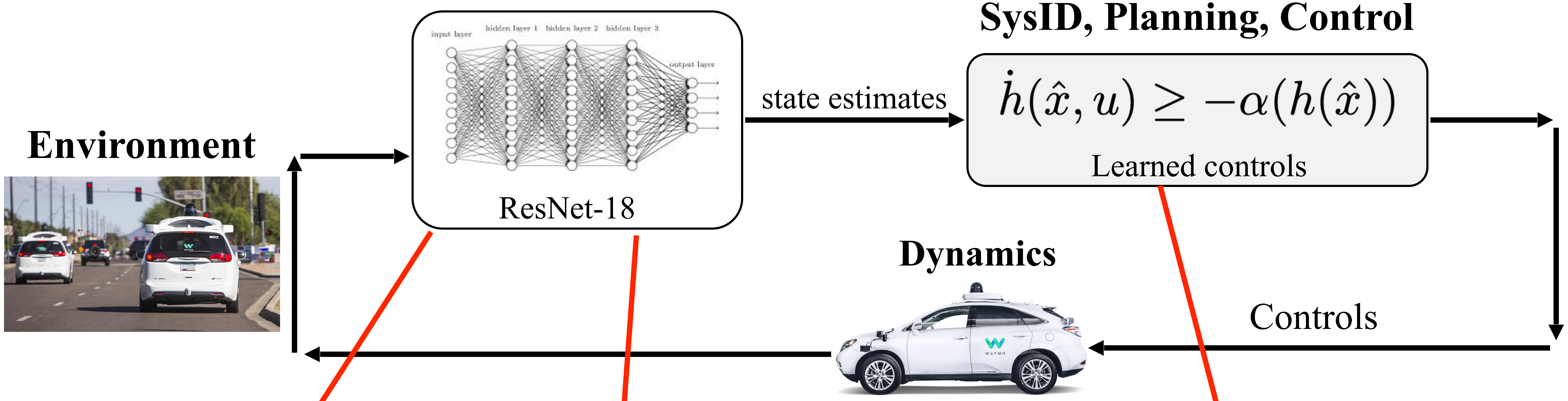
$$\dot{h}(\hat{x}, u) \geq -\alpha(h(\hat{x}))$$

Learned controls

Controls

Dynamics

Safe control for learning-enabled autonomy



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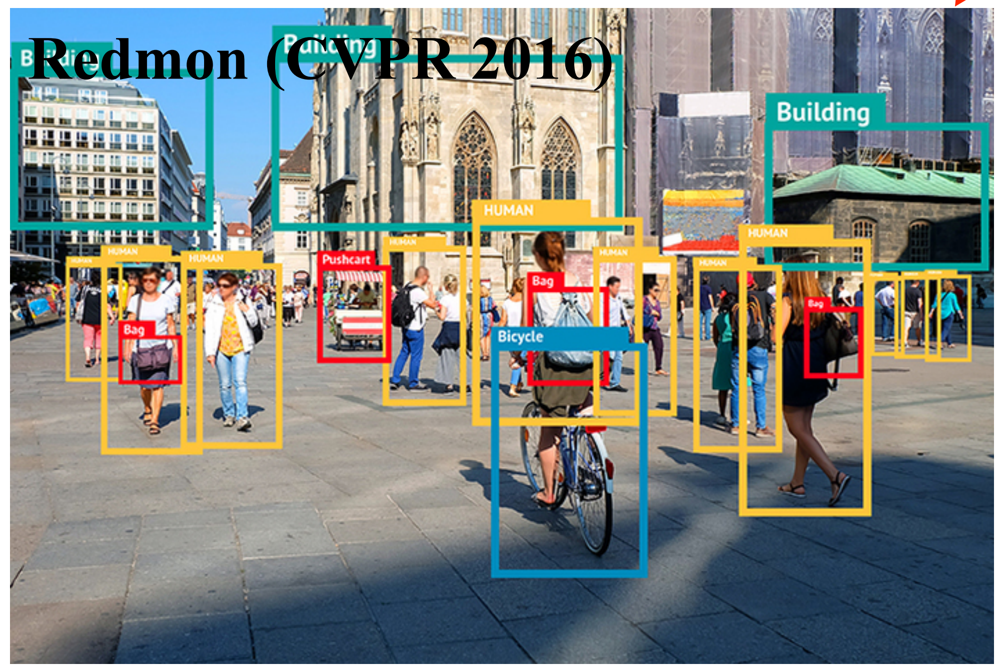
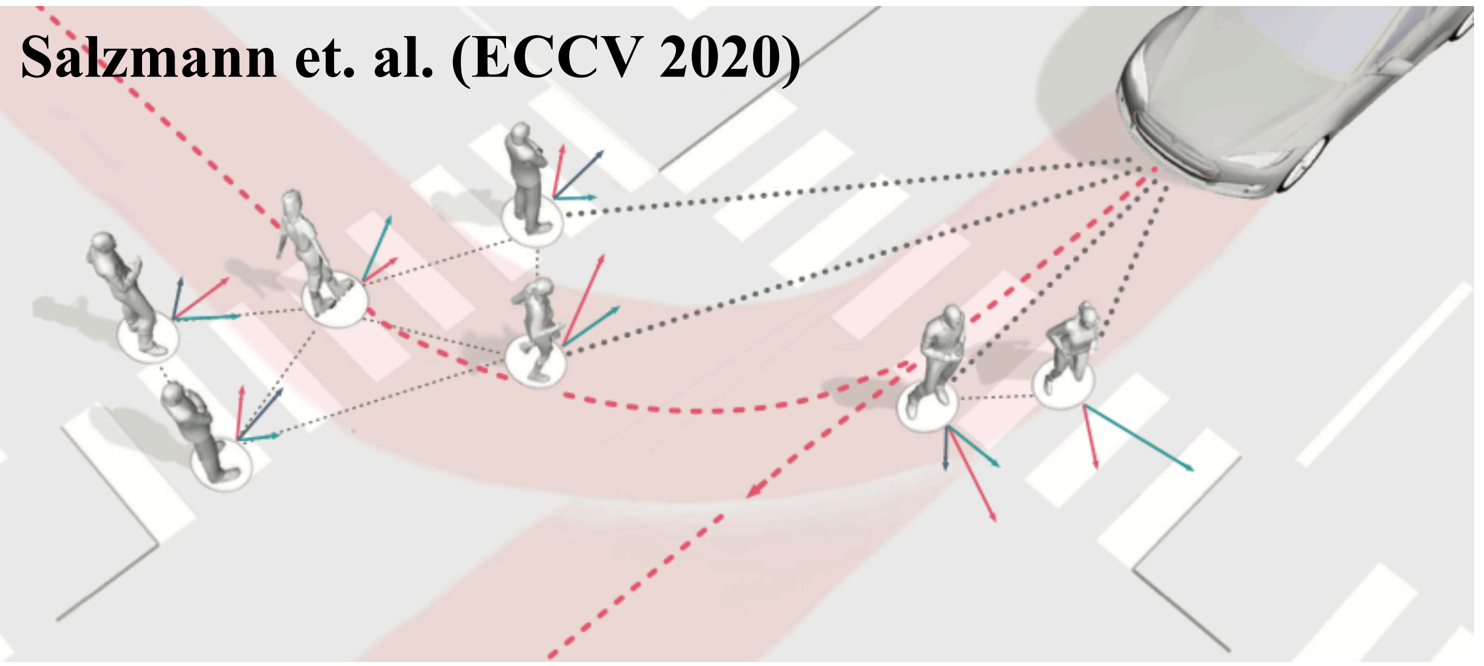
Learning-enabled components:



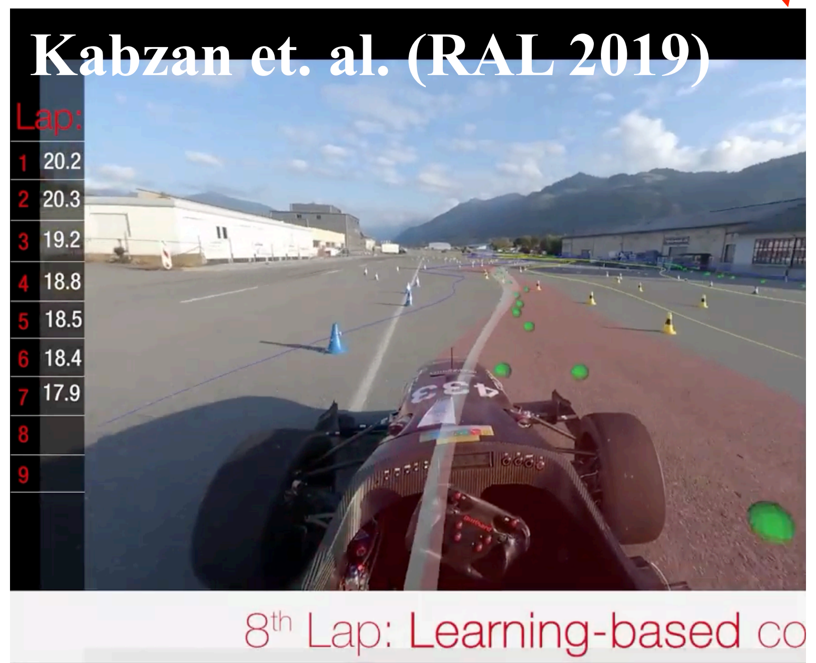
Redmon (CVPR 2016)

Object detection



Salzmann et. al. (ECCV 2020)

Prediction

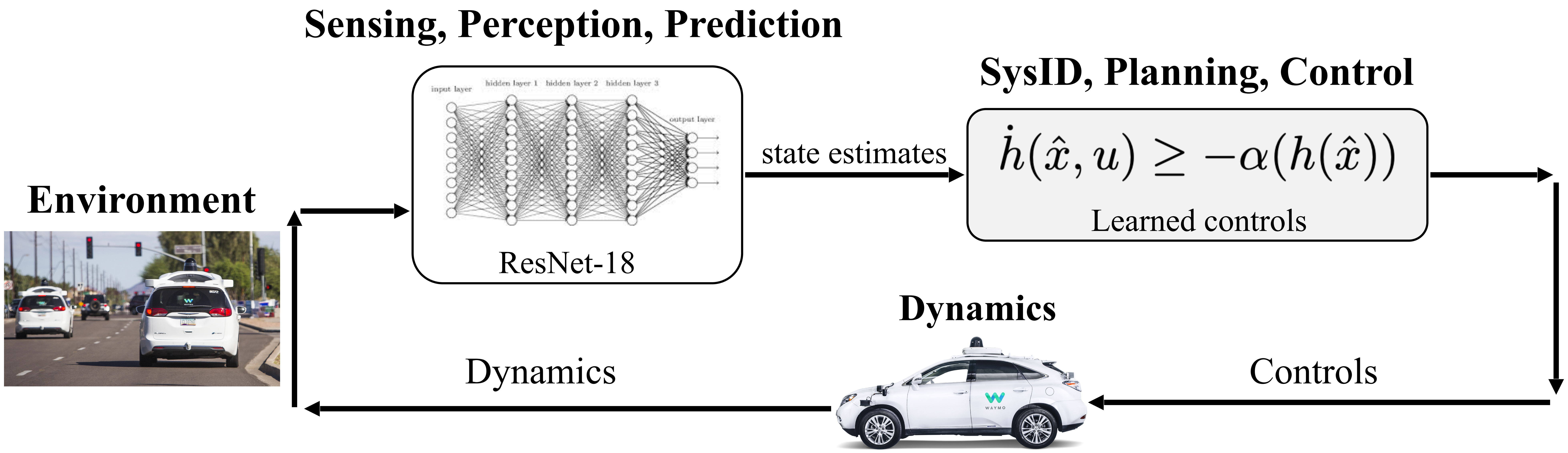


Kabzan et. al. (RAL 2019)

Lindemann et. al. (OJCSYS 2024)

Modelling and control

Safe control for learning-enabled autonomy



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Core challenges:

- Complex environments and sensors
- Learning-enabled components
- Learning-enabled multi-agent systems

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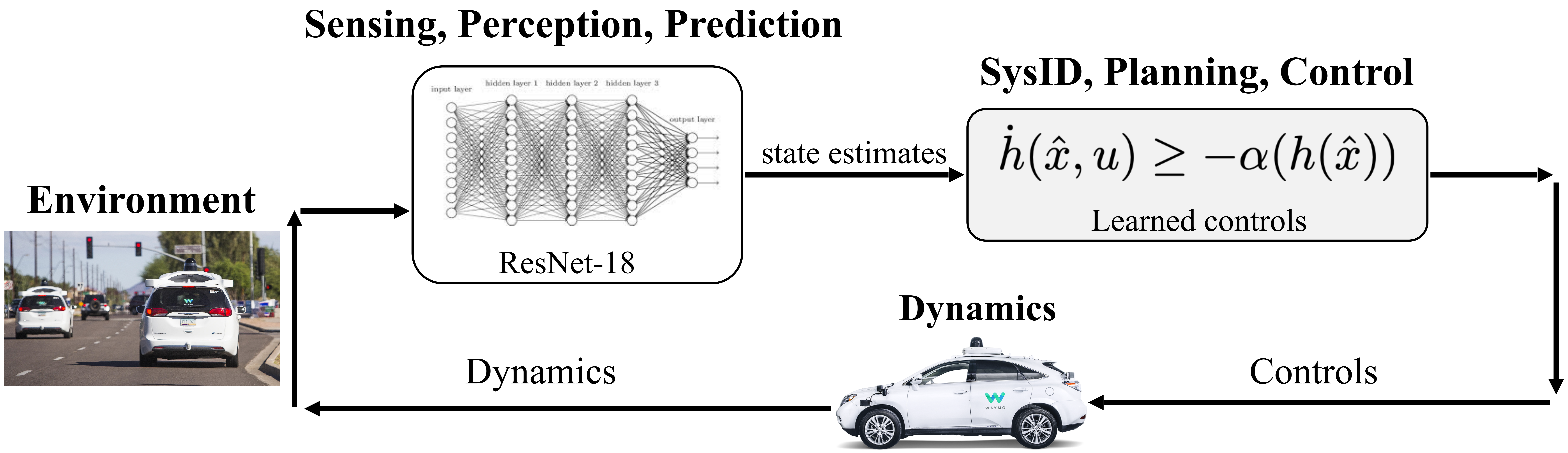

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Learned controls

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Dynamics

Dynamics

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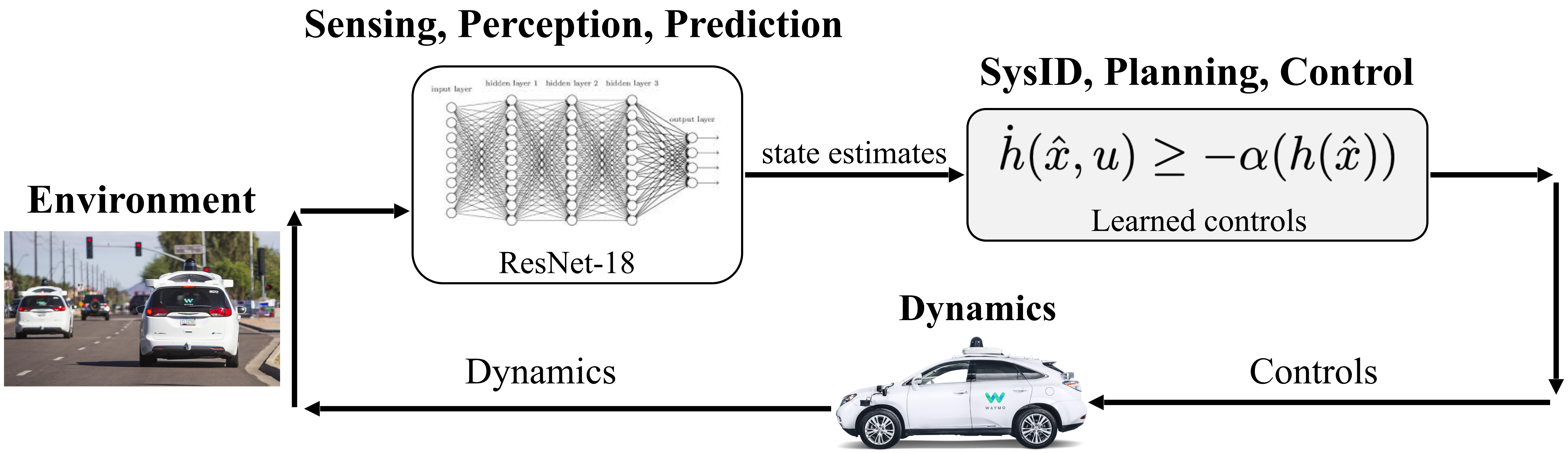
Core challenges:

- Complex environments and sensors ← **Uncertainty-aware, reactive control**
- Learning-enabled components ← **Robustness and (online) verification**
- Learning-enabled multi-agent systems ← **Distributed control**

Safe control for learning-enabled autonomy



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Learned controls

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Dynamics

Dynamics

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Core challenges:

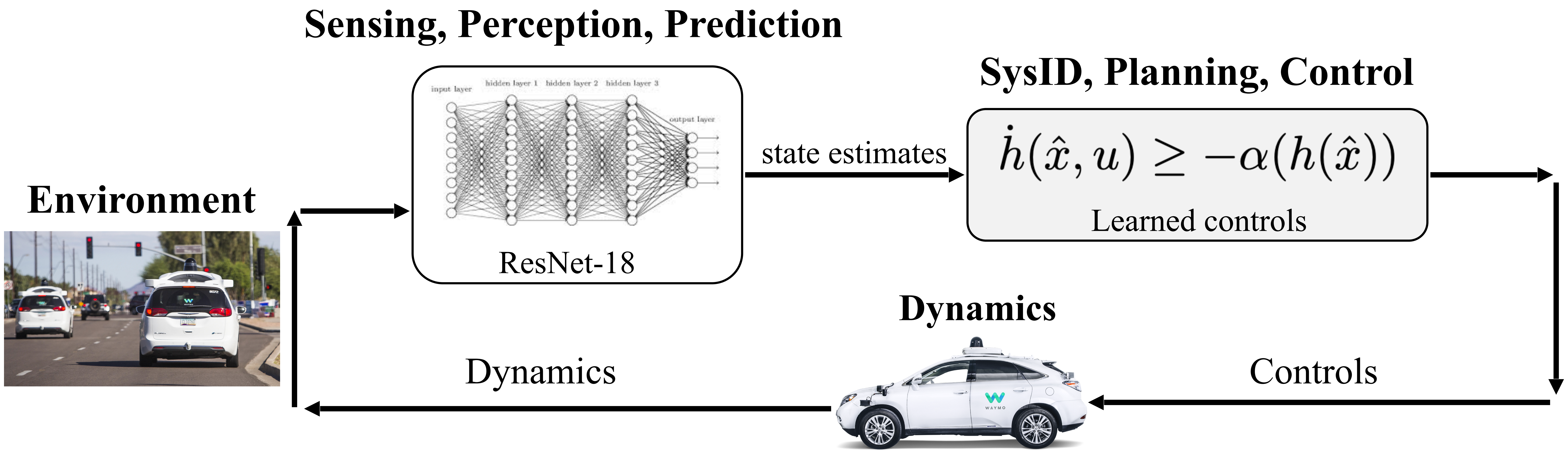
- Complex environments and sensors
- Learning-enabled components
- Learning-enabled multi-agent systems
- System specifications
- Simulators, models, and data
- Real-time computation

Uncertainty-aware, reactive control

Robustness and (online) verification

Distributed control

Safe control for learning-enabled autonomy



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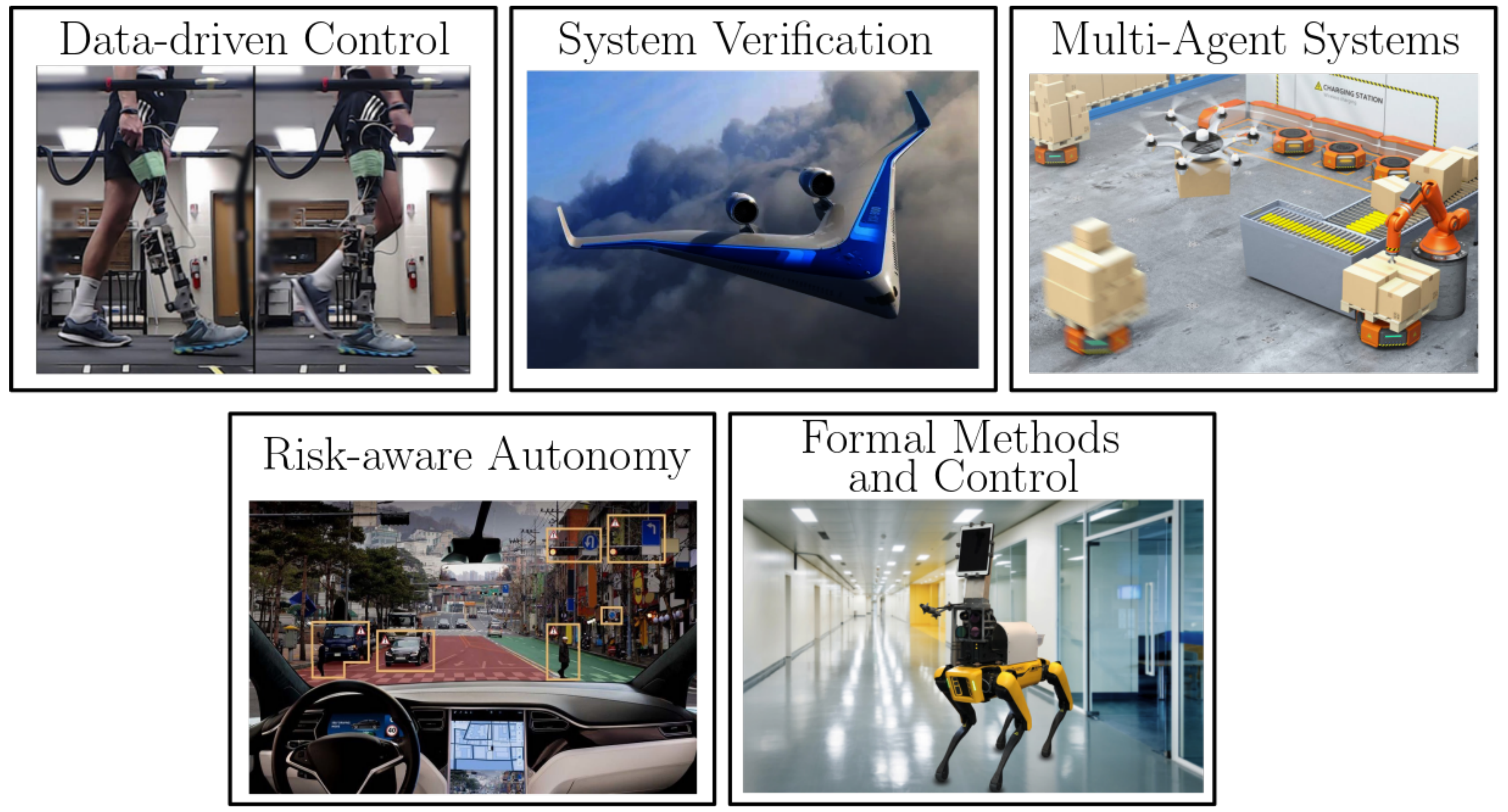
Core challenges:

- Complex environments and sensors ← **Uncertainty-aware, reactive control**
- Learning-enabled components ← **Robustness and (online) verification**
- Learning-enabled multi-agent systems ← **Distributed control**
- System specifications ← **Dynamic formal languages under uncertainty**
- Simulators, models, and data ← **Mitigating Sim2Real Gap**
- Real-time computation ← **Efficient computational algorithms**

My lab’s research agenda

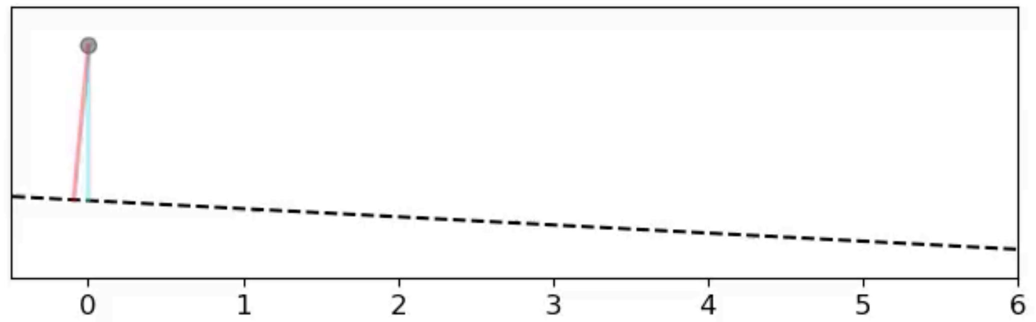
Verifiable control frameworks for autonomy along with **efficient computational algorithms**.

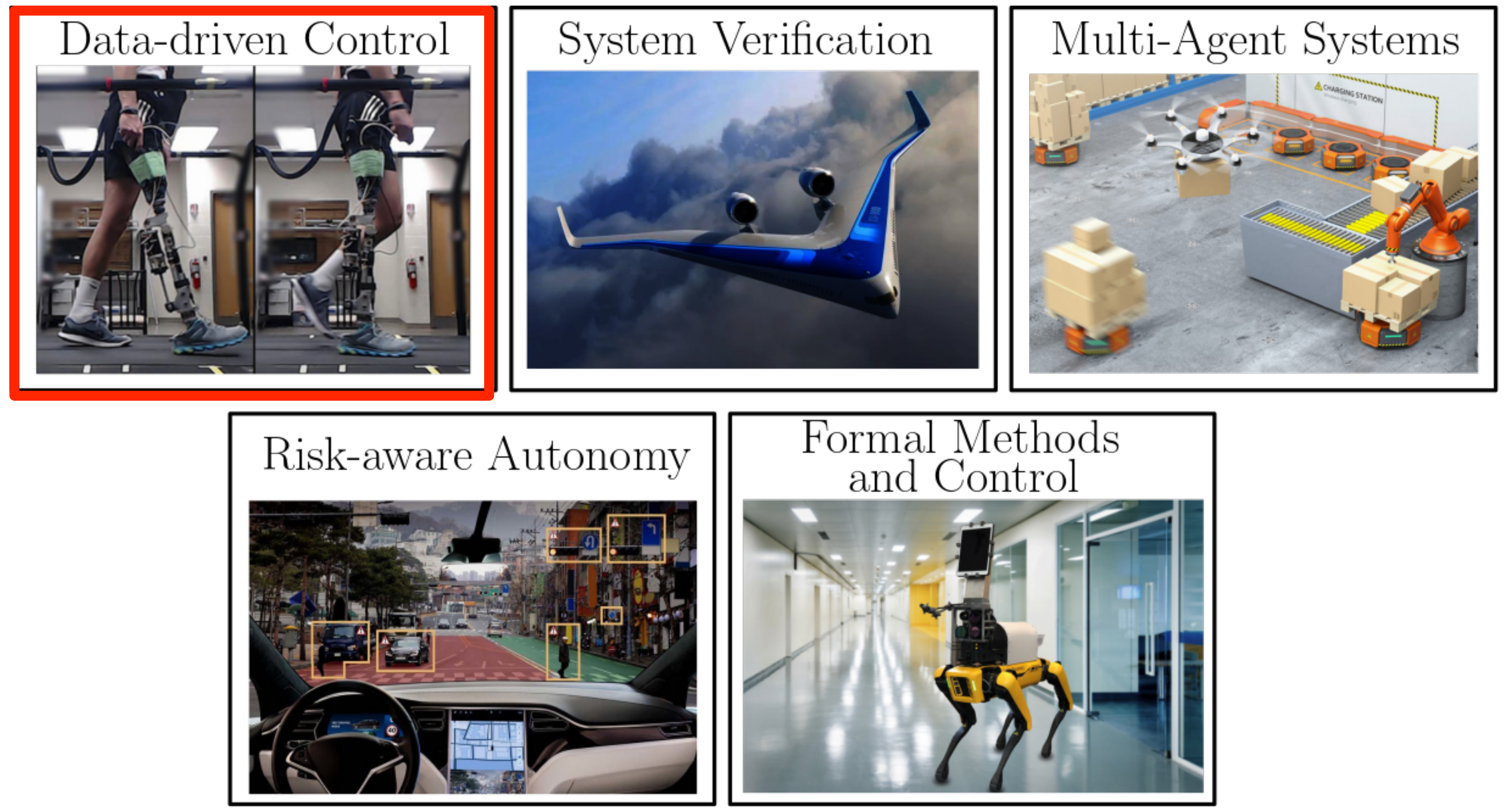


My lab’s research agenda

Verifiable control frameworks for autonomy along with **efficient computational algorithms**.

Learning control laws from demonstrations





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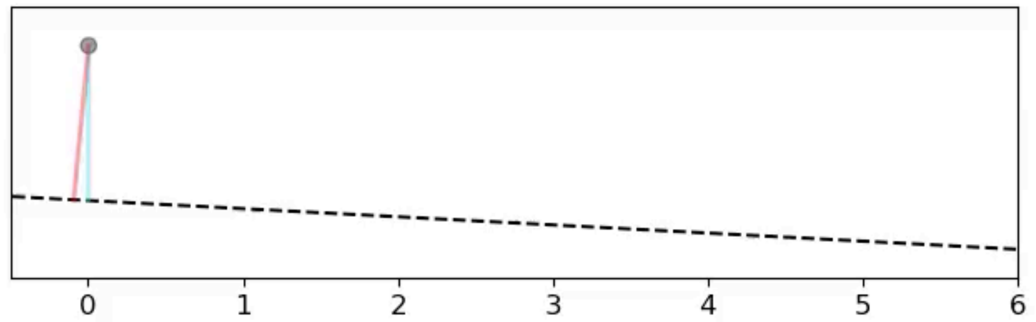
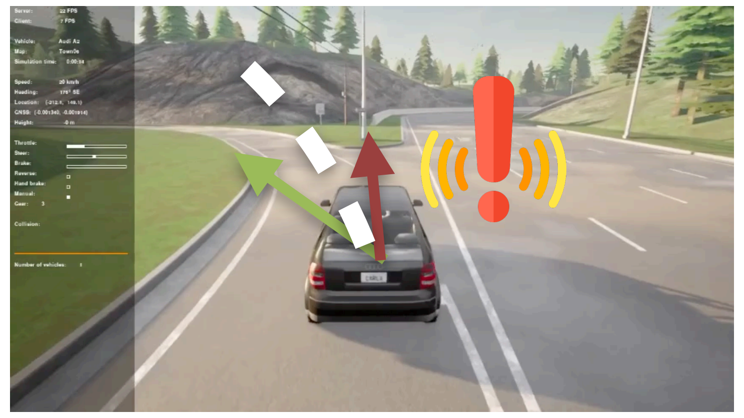
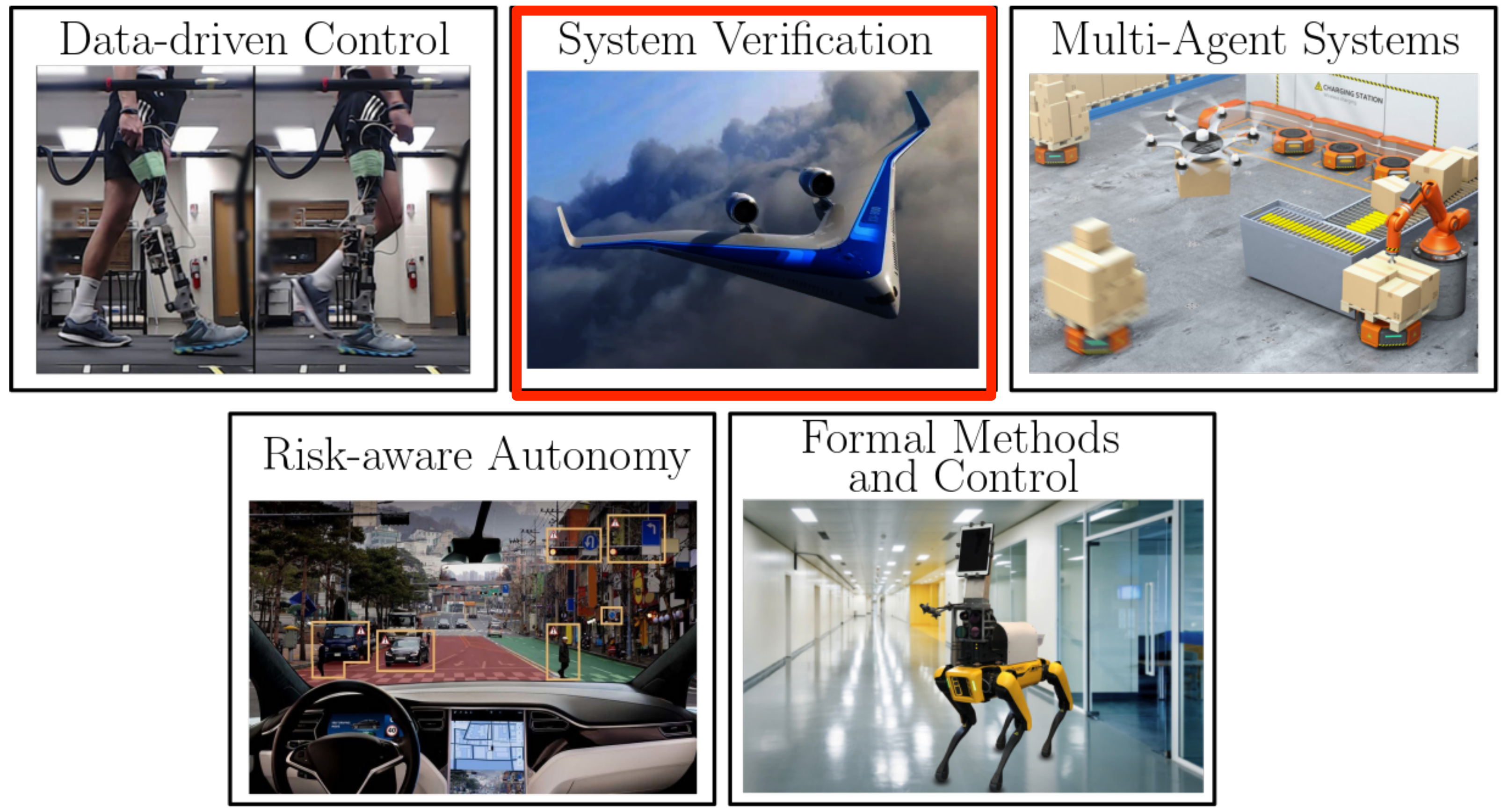


Statistical system verification





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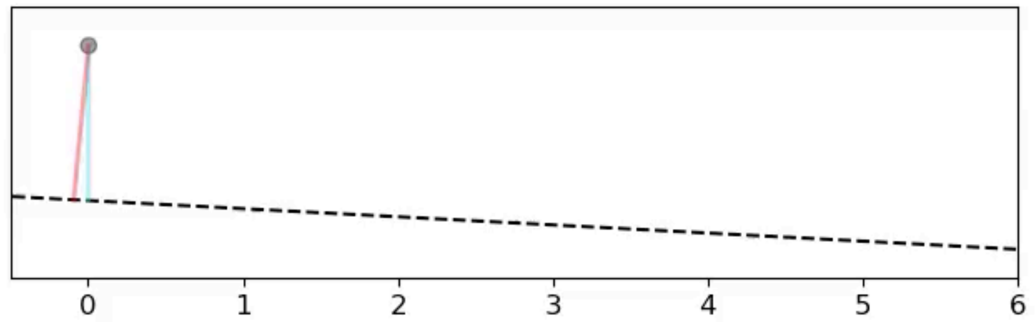

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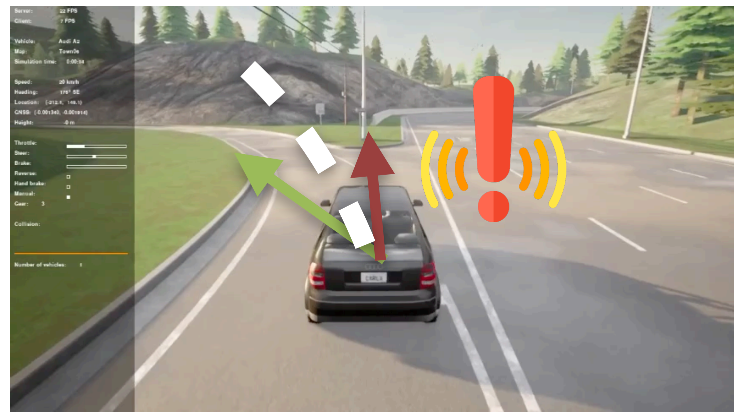
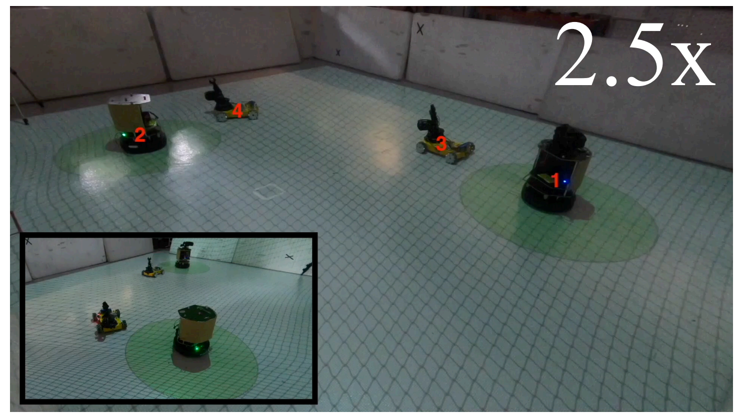
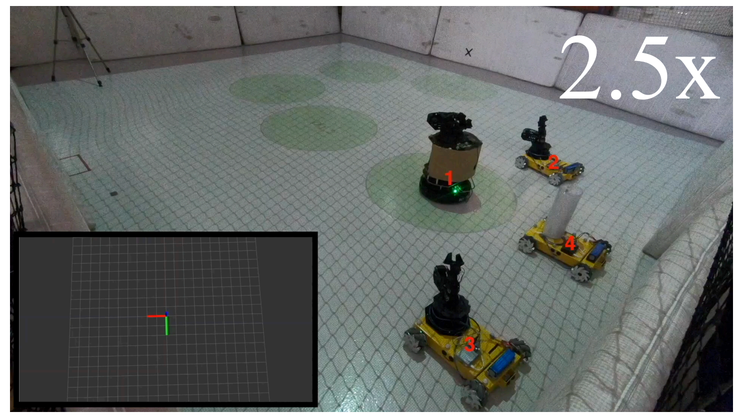
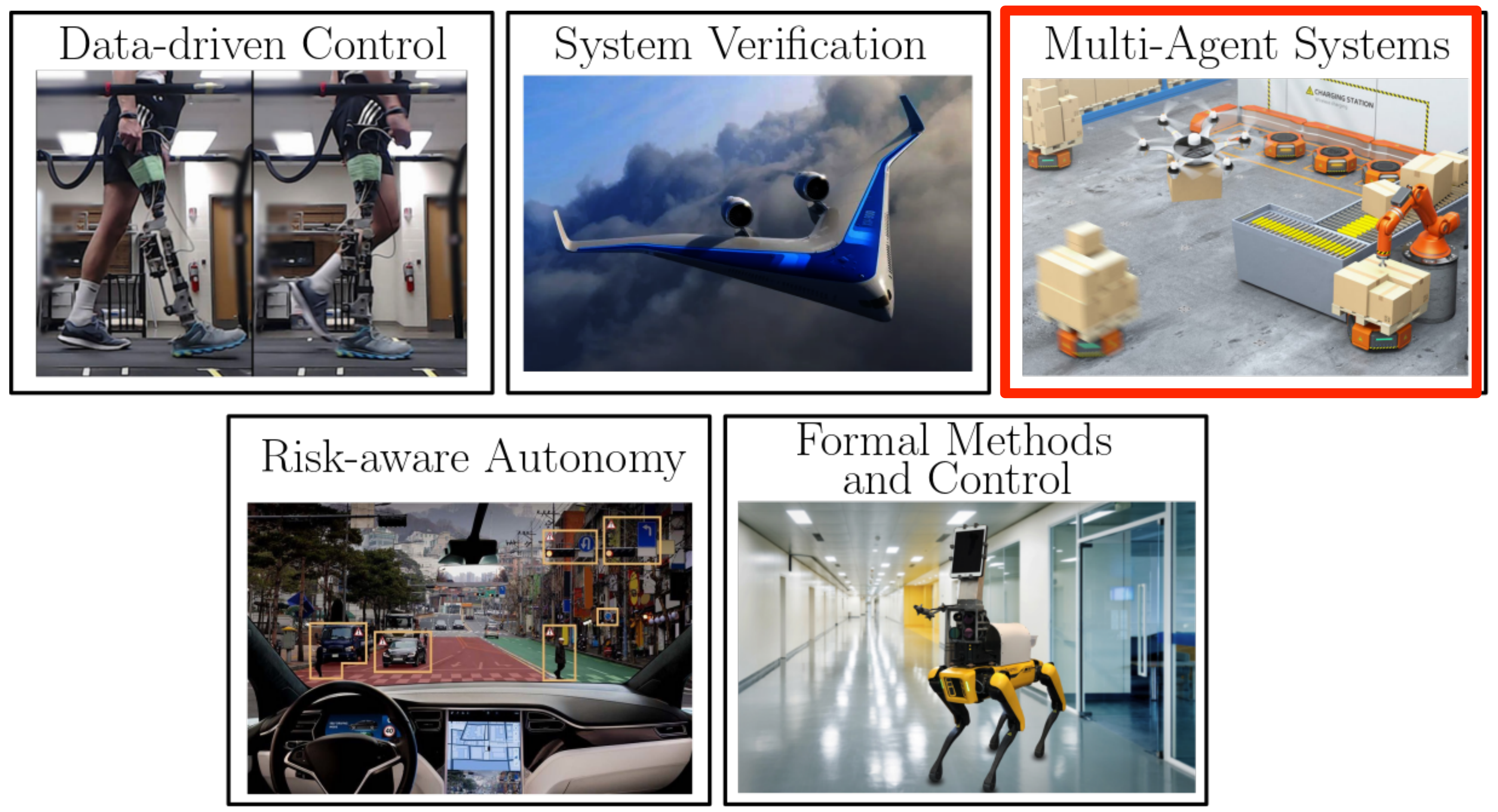


Temporal logic-constrained multi-agent systems







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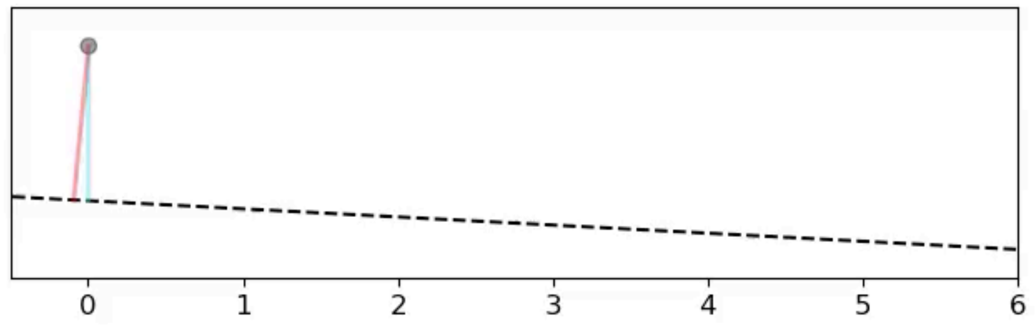

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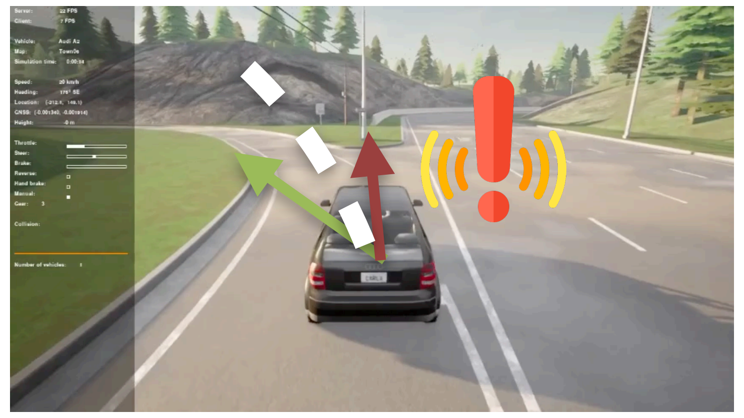

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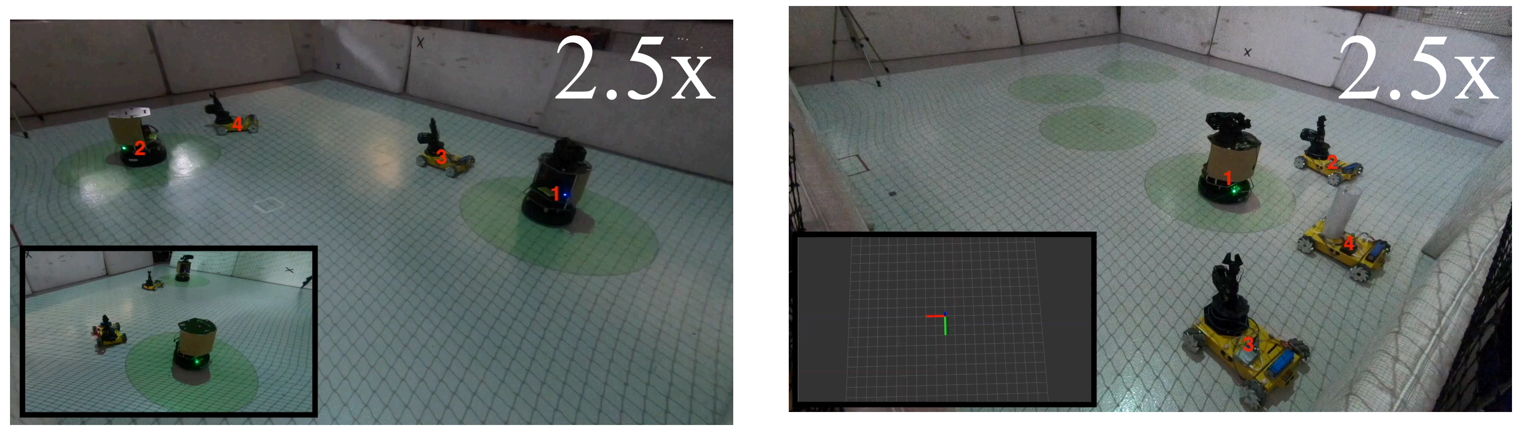
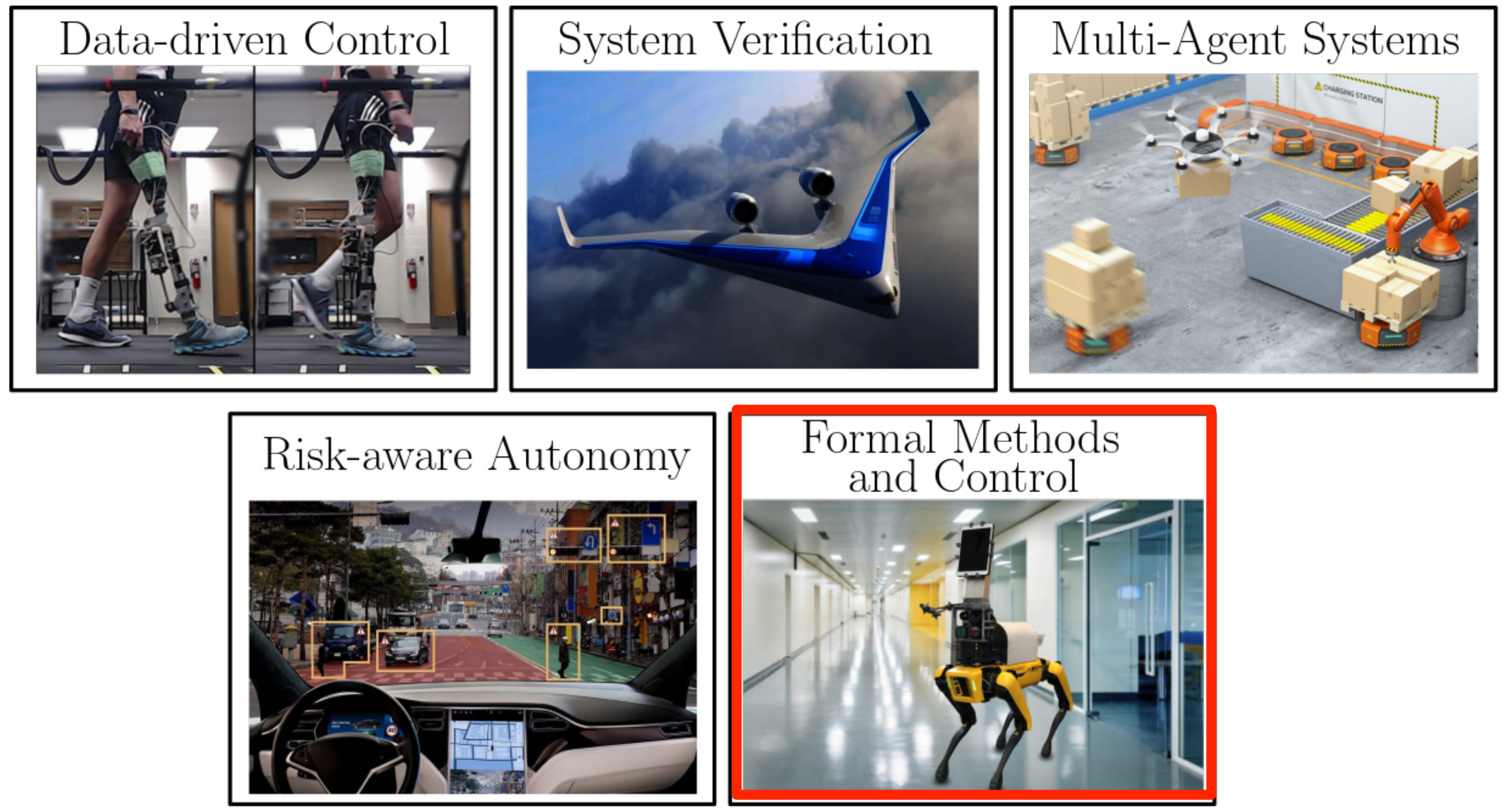
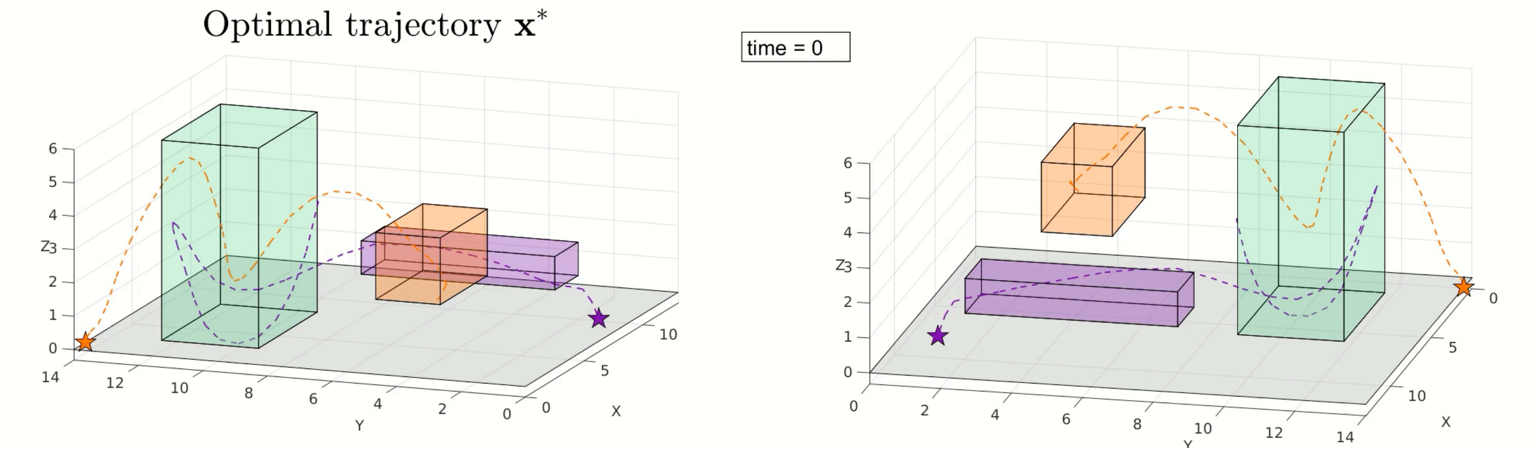




Formal methods-based control



temporally robust control

My lab’s research agenda

Verifiable control frameworks for autonomy along with **efficient computational algorithms**.

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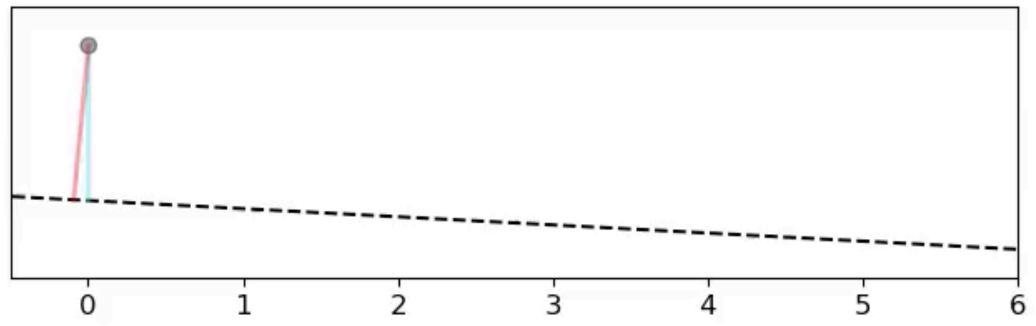

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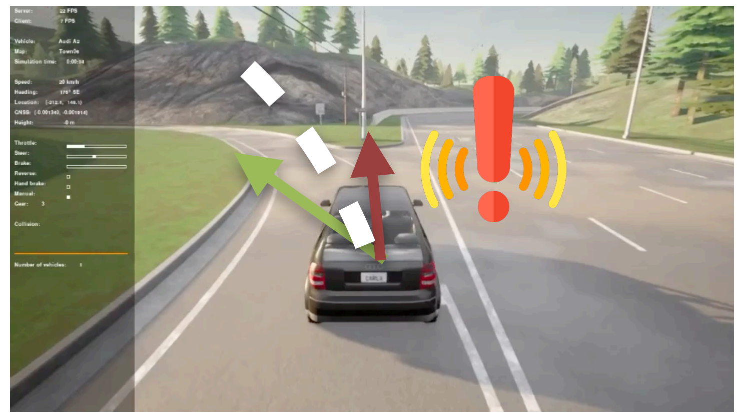

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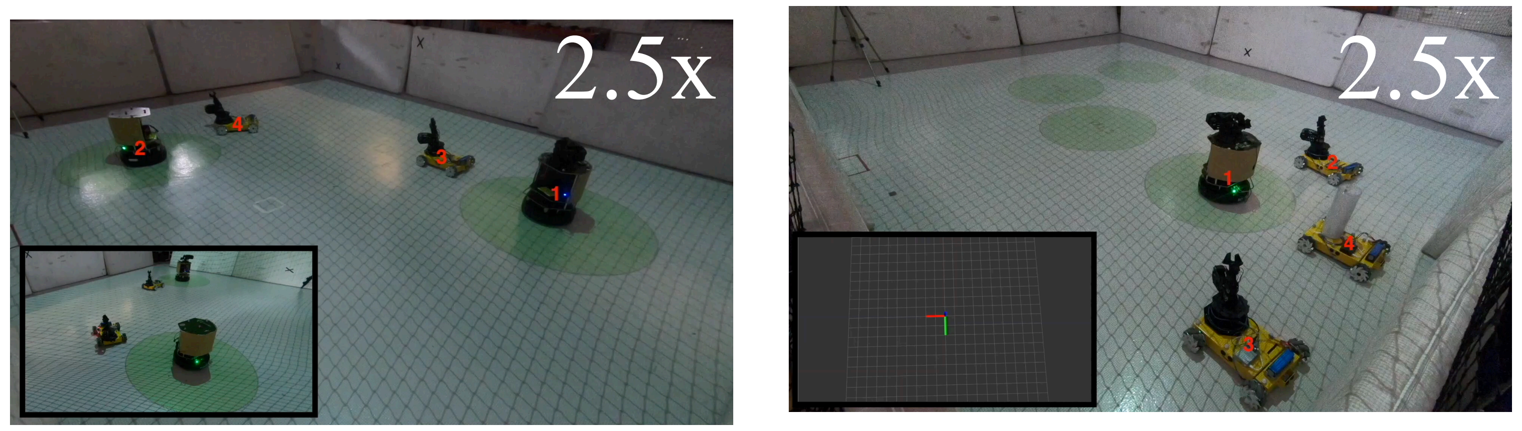
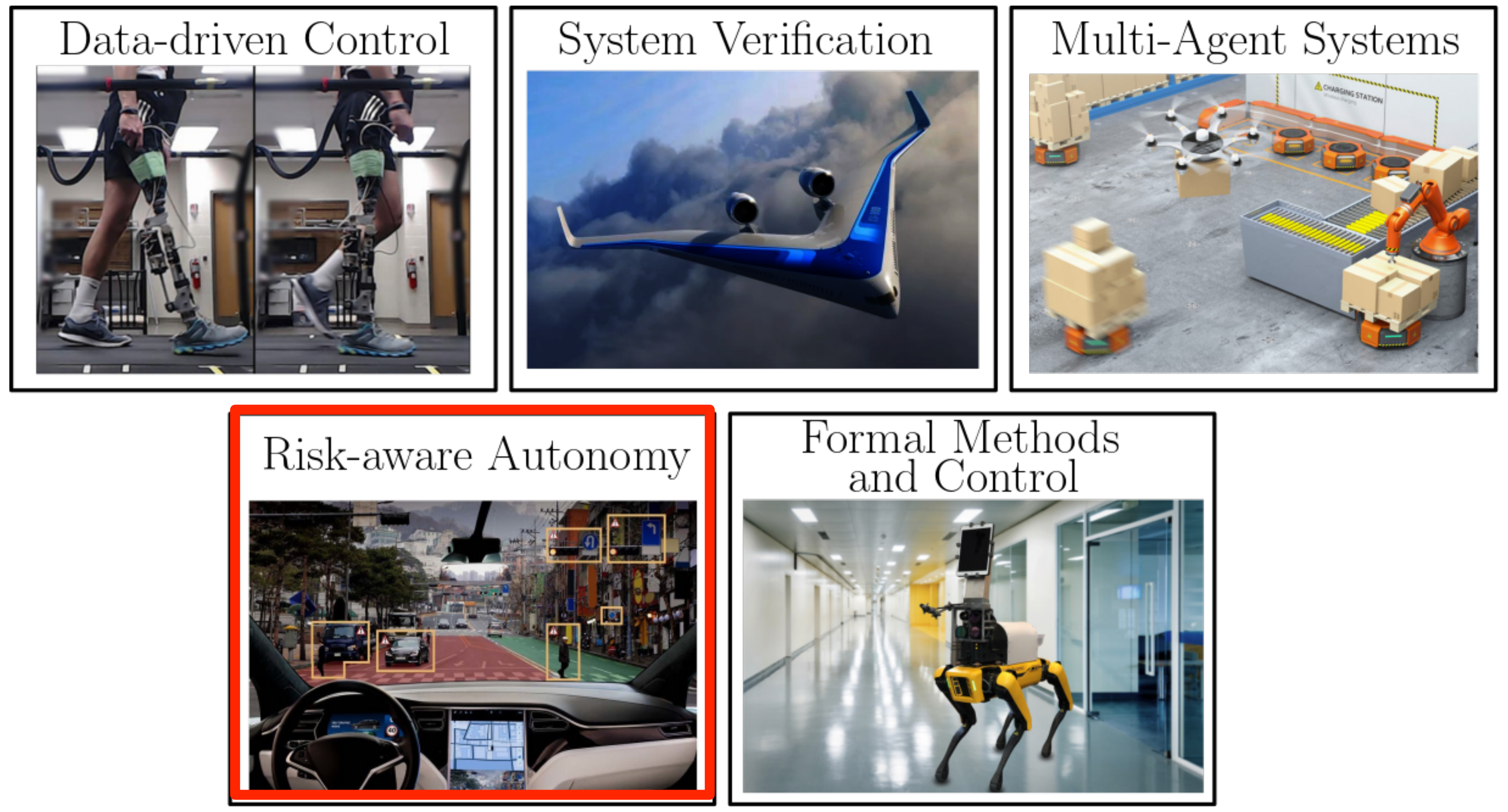


Decision making under uncertainty



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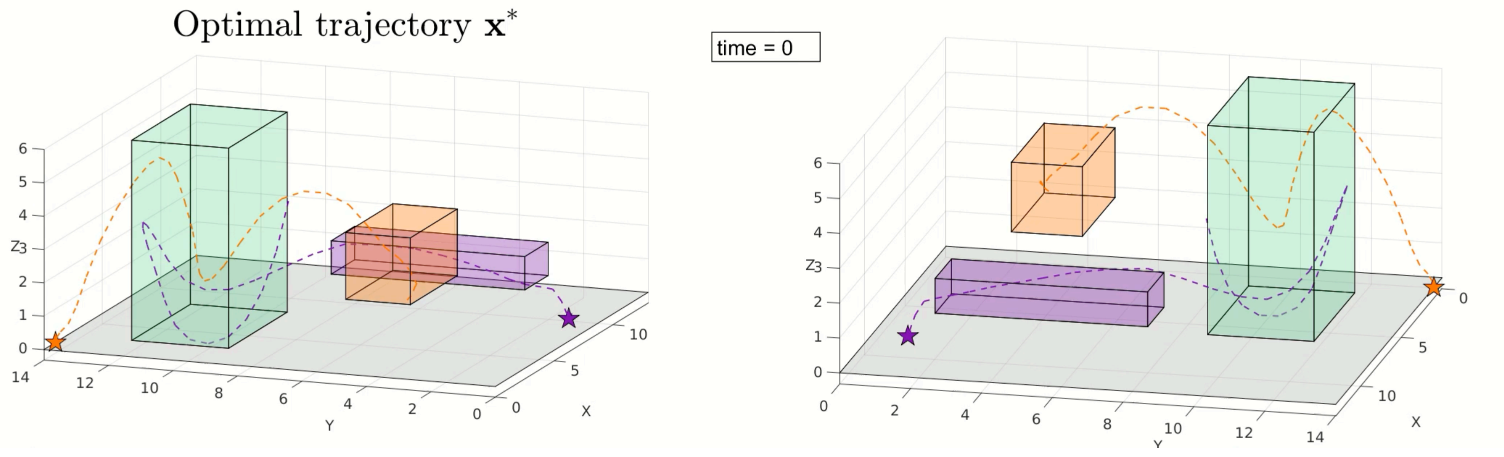

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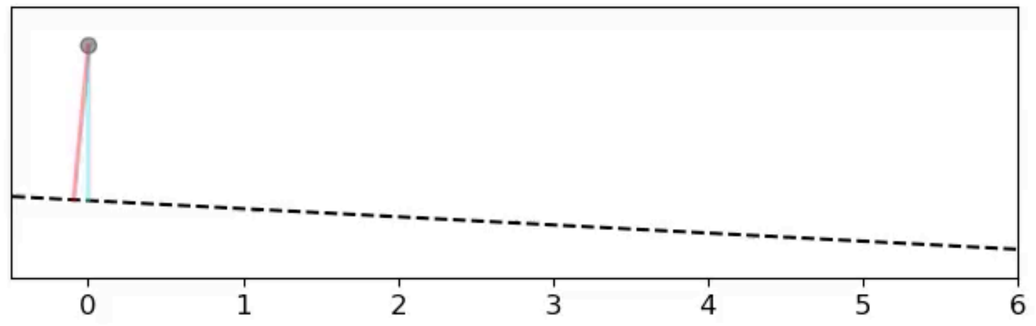

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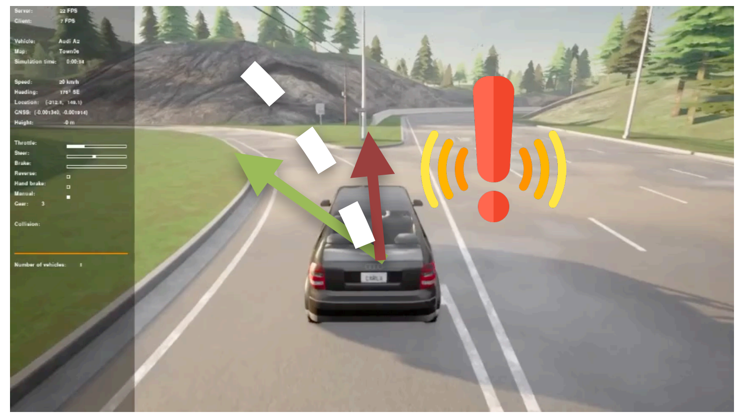

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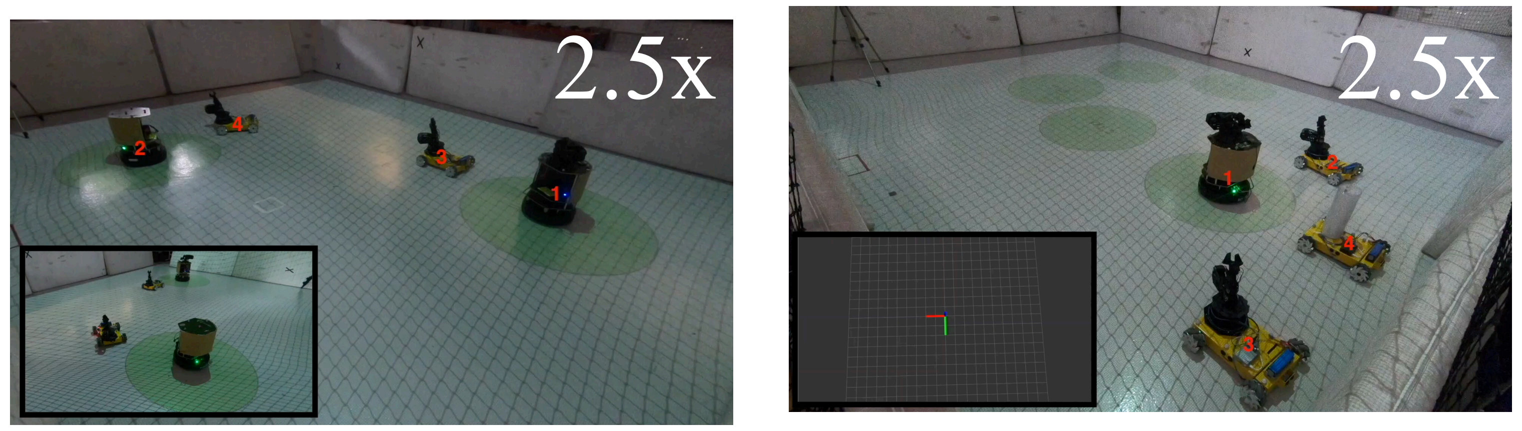

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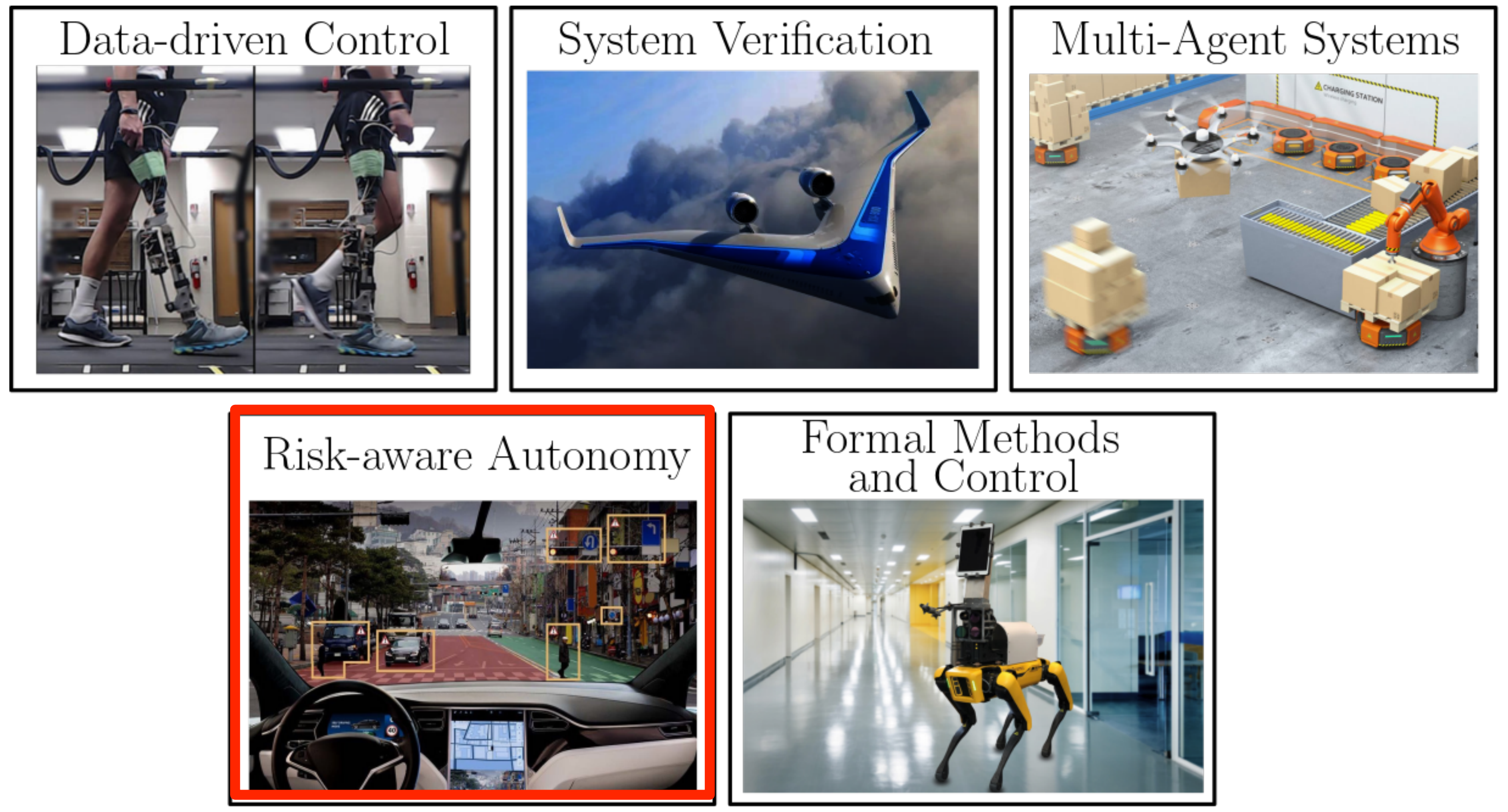

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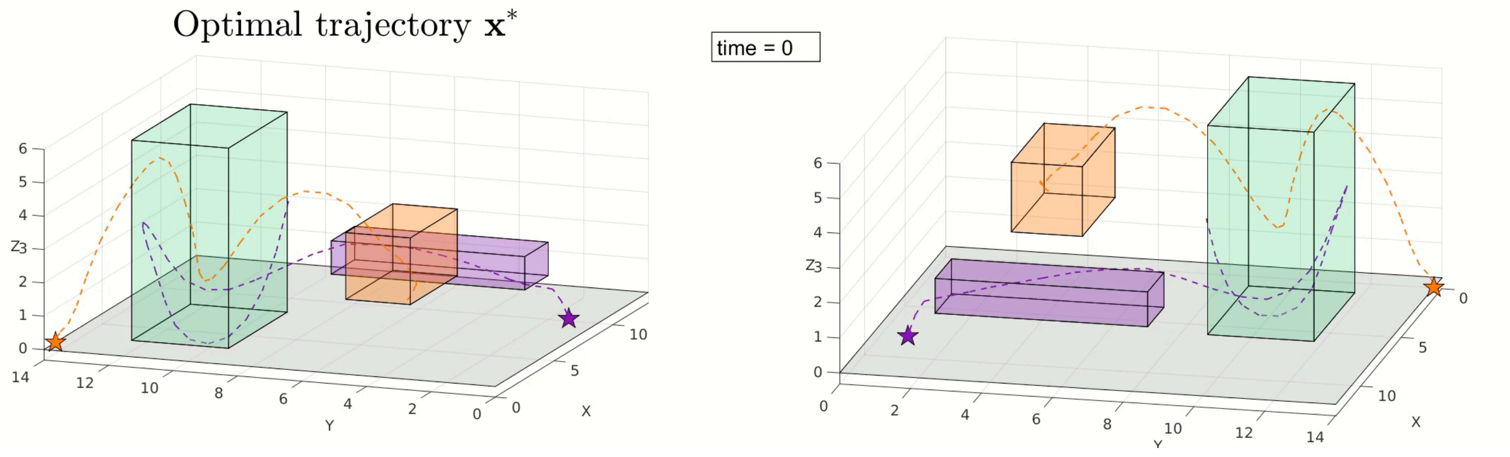
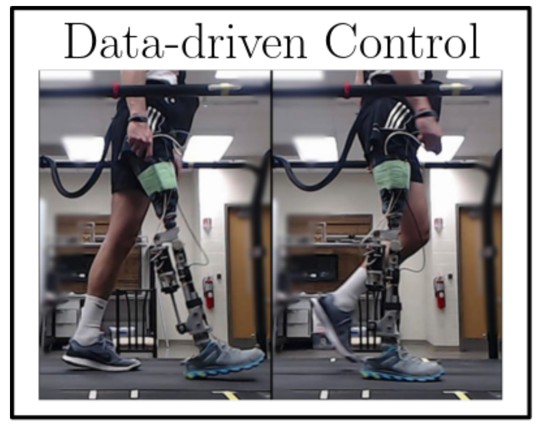


temporally robust control

My research provides a **formal and computational approach** to systems and control theory.

Safe Control in Dynamic Environments with Conformal Prediction


Data-driven Control

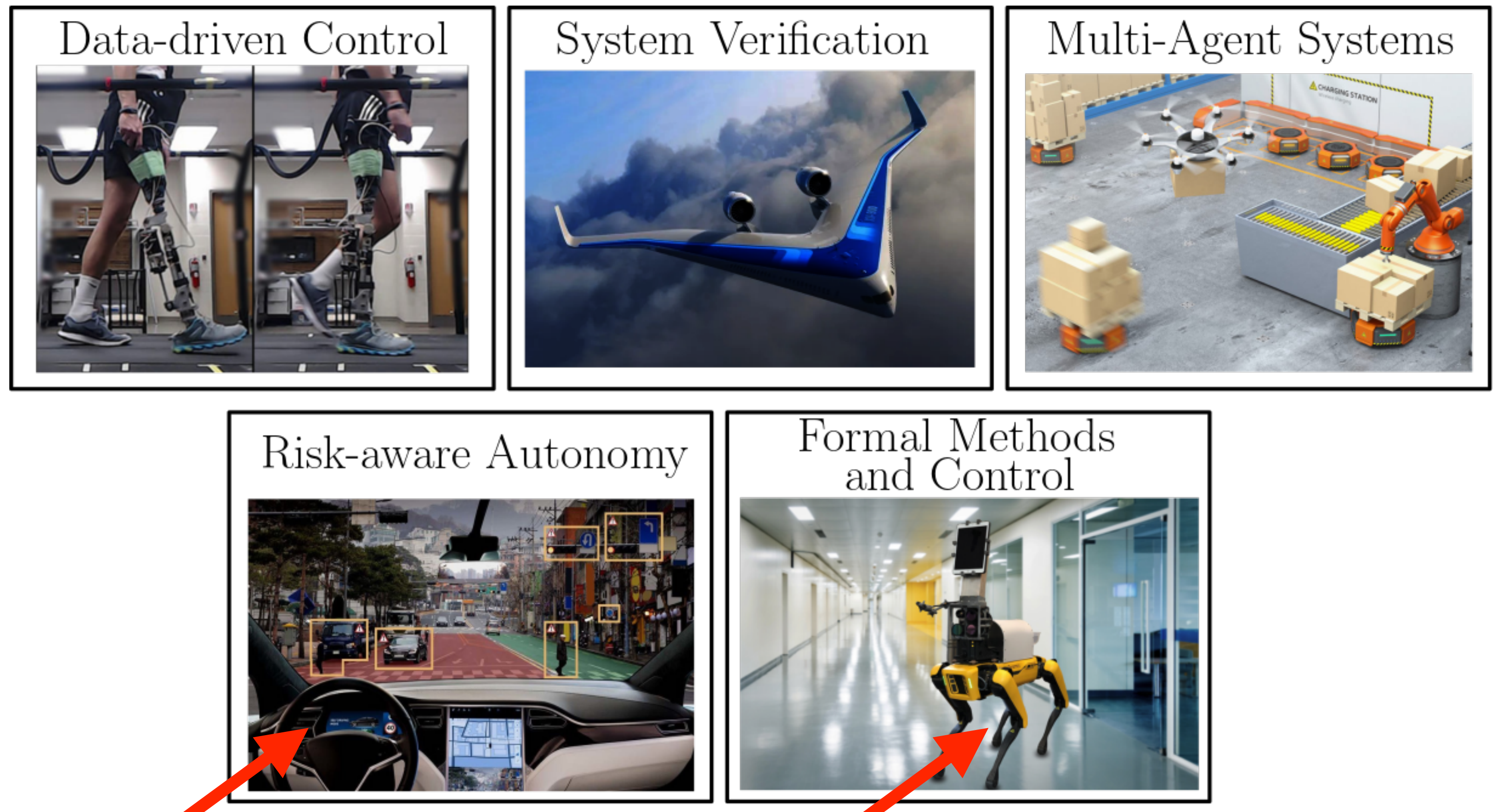

System Verification


Multi-Agent Systems


Risk-aware Autonomy


Formal Methods and Control

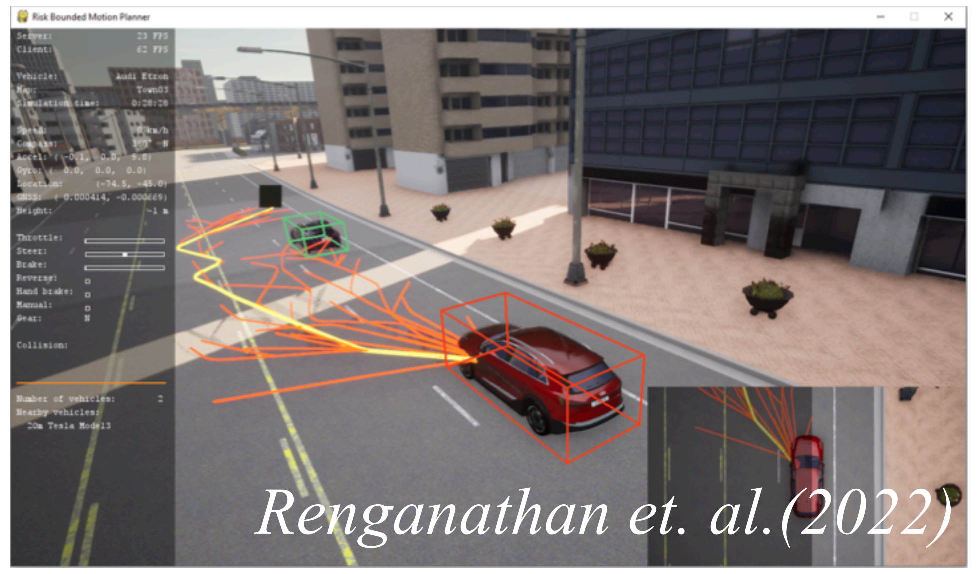
Safe Control in Dynamic Environments with Conformal Prediction



Today, with a little bit of …
(~ last 2 years)

The safe control problem in dynamic environments

Compute **control inputs** so that the system avoids **dynamic agents** with a probability of at least $1-\delta$.


Renganathan et. al.(2022)

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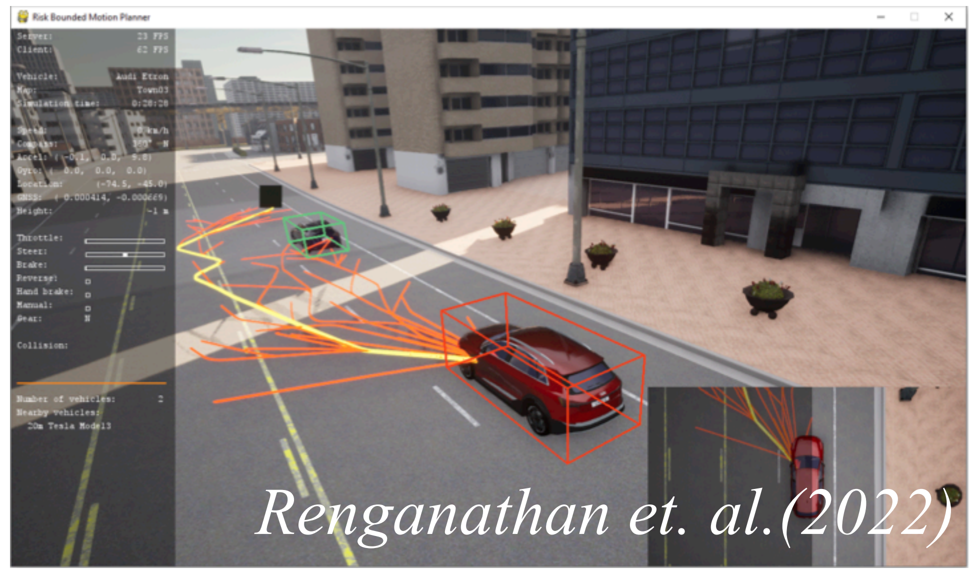
Challenges:

- Stochastic and dynamic agents
- Complex learning-enabled predictors



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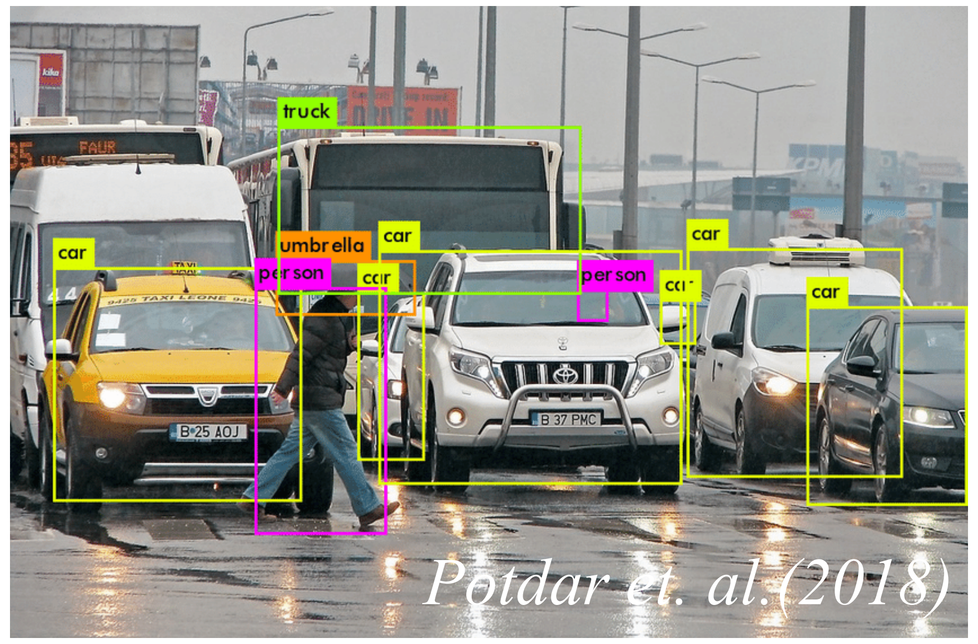

Potdar et. al.(2018)

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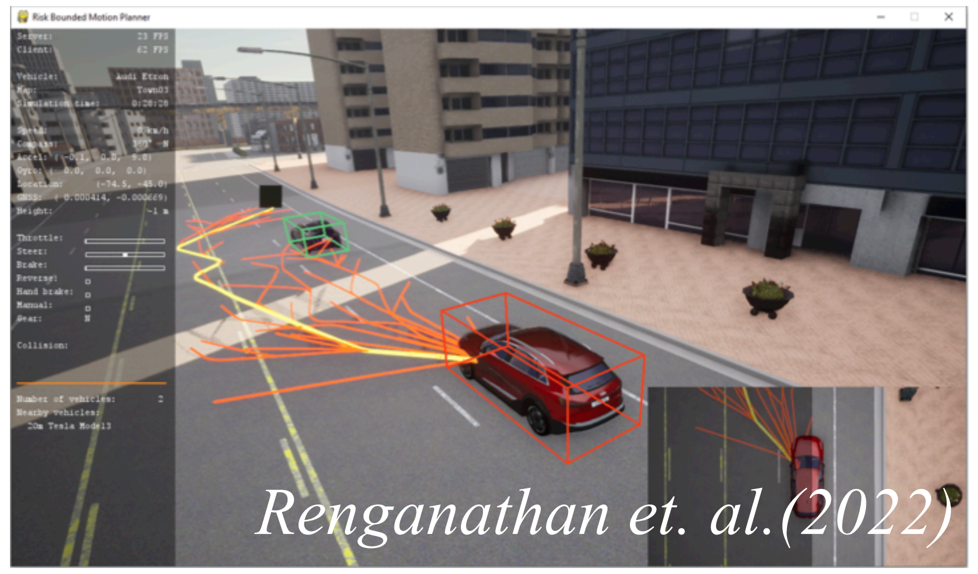
uncertainty quantification within control difficult



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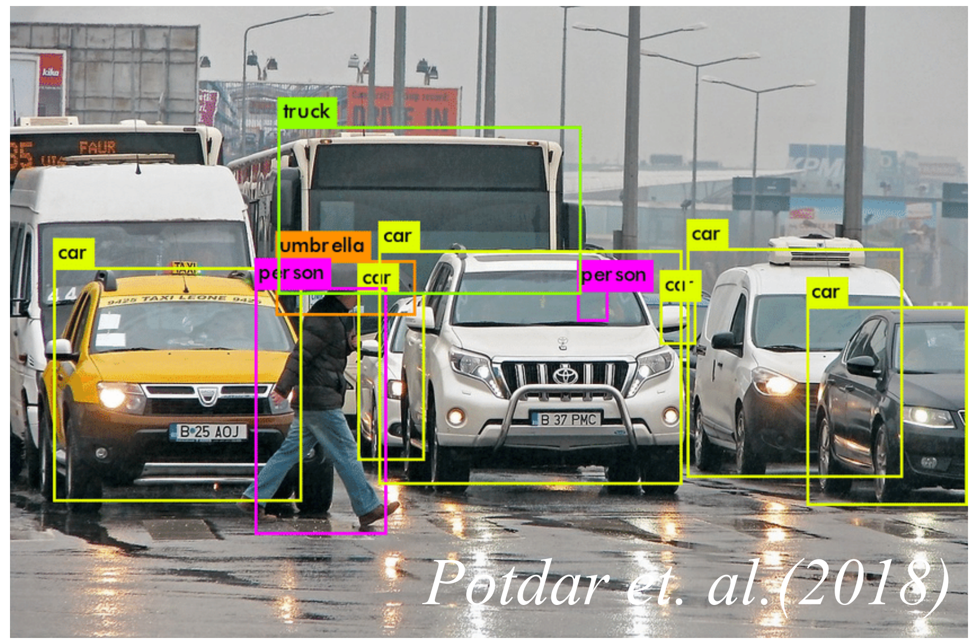

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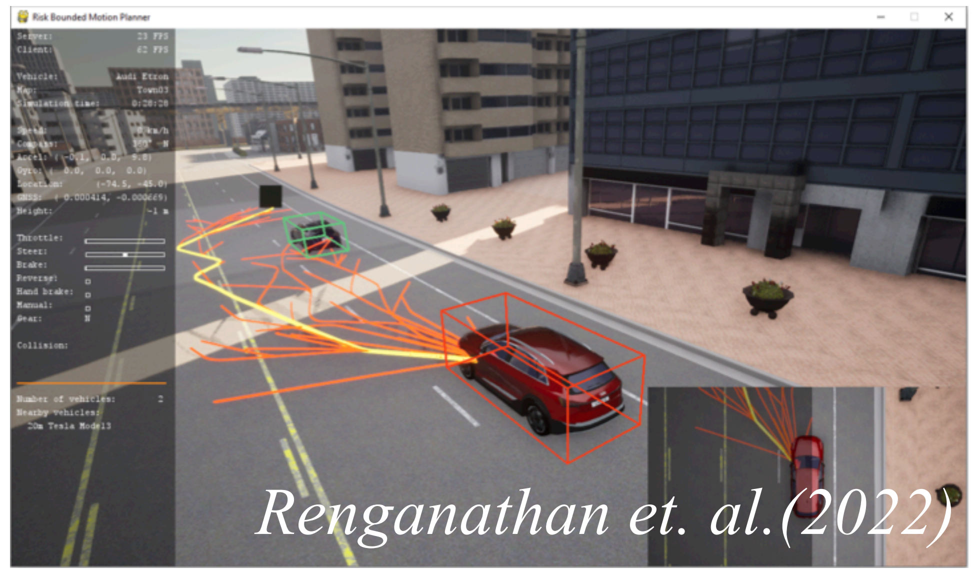
Existing work (references omitted):

- “Vanilla” motion planner (e.g., graph/sampling-based)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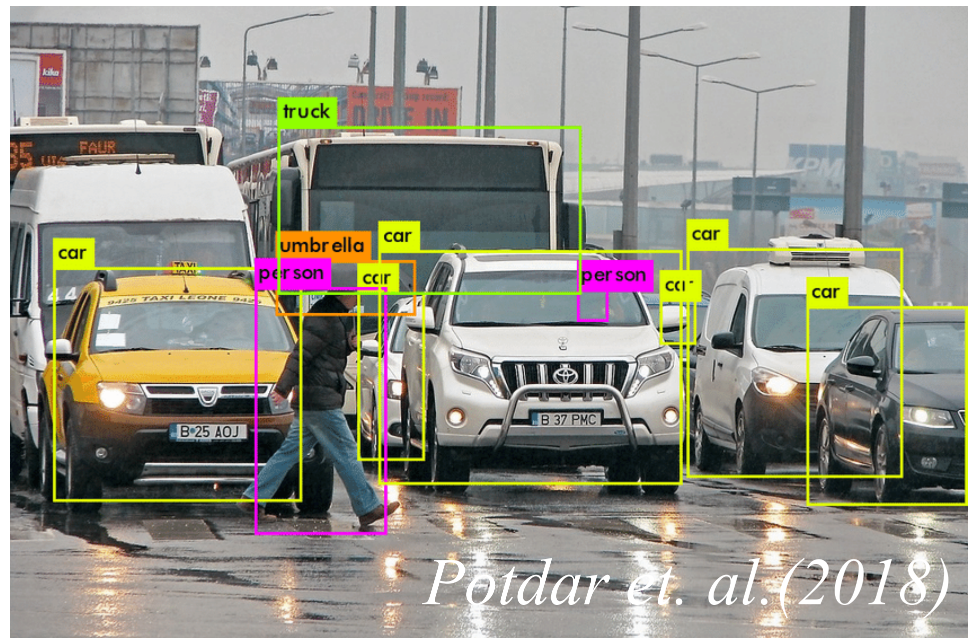

Potdar et. al.(2018)

The safe control problem in dynamic environments

Compute **control inputs** so that the system avoids **dynamic agents** with a probability of at least $1 - \delta$.

Challenges:

- Stochastic and dynamic agents
- Complex learning-enabled predictors

uncertainty quantification within control difficult

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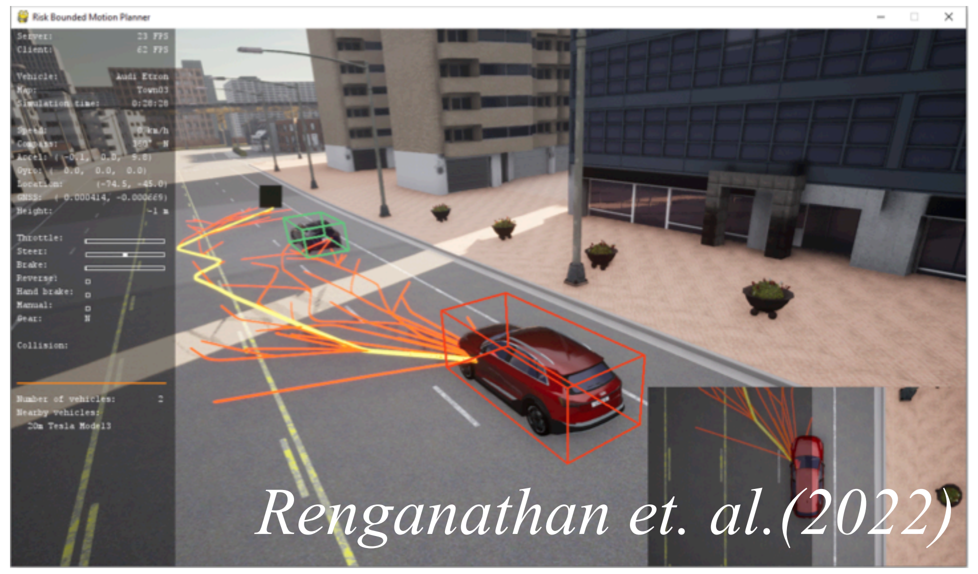

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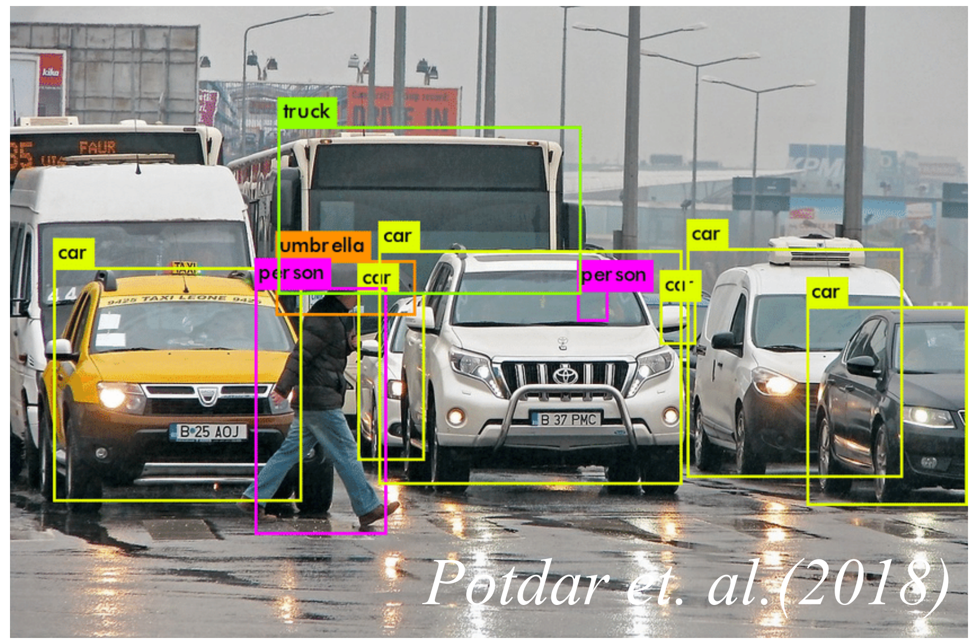

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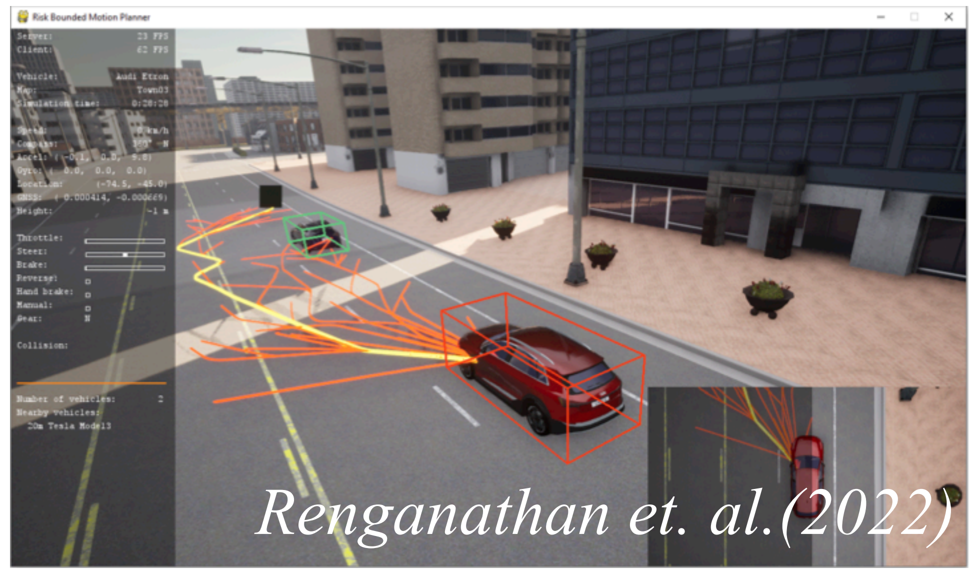

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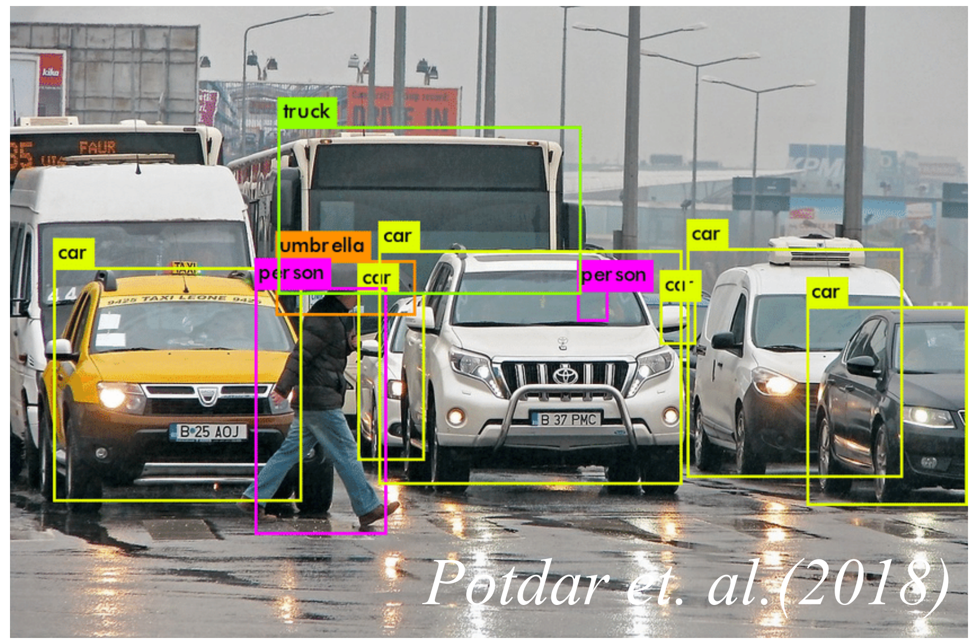

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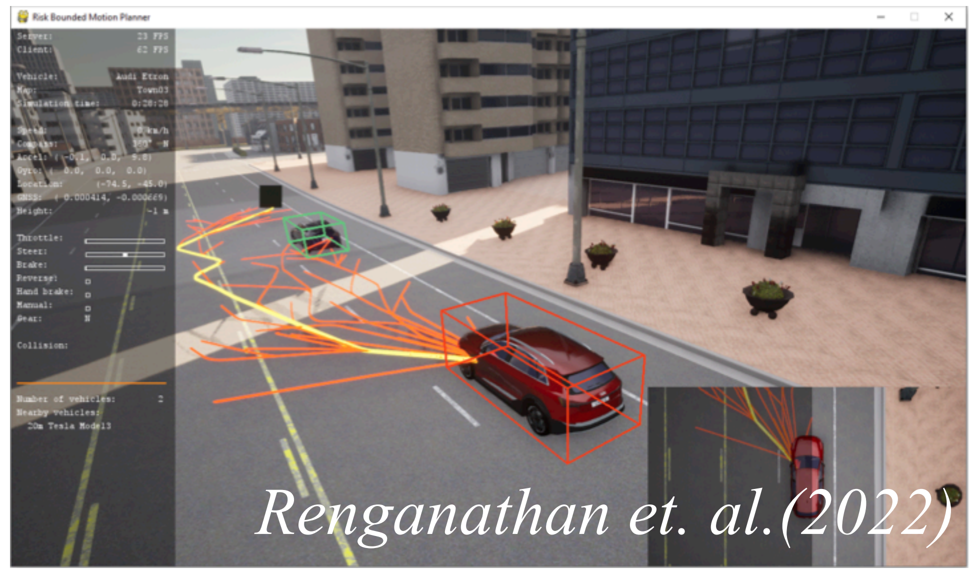

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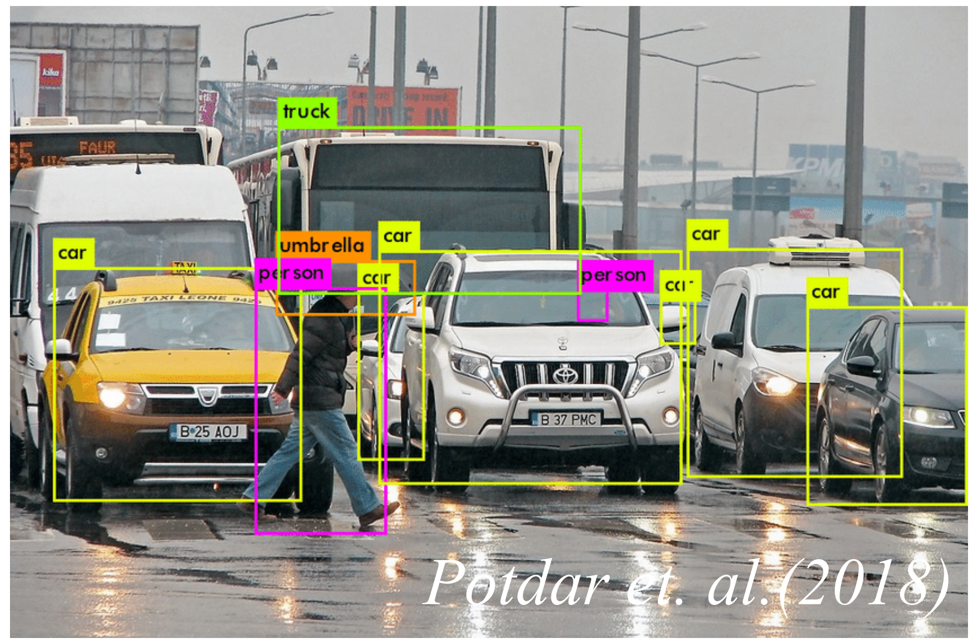

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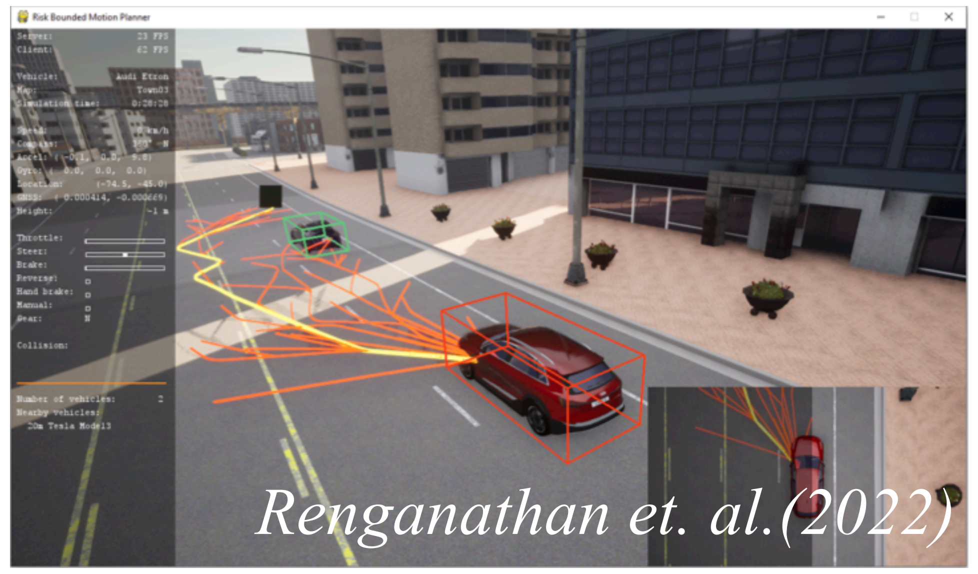
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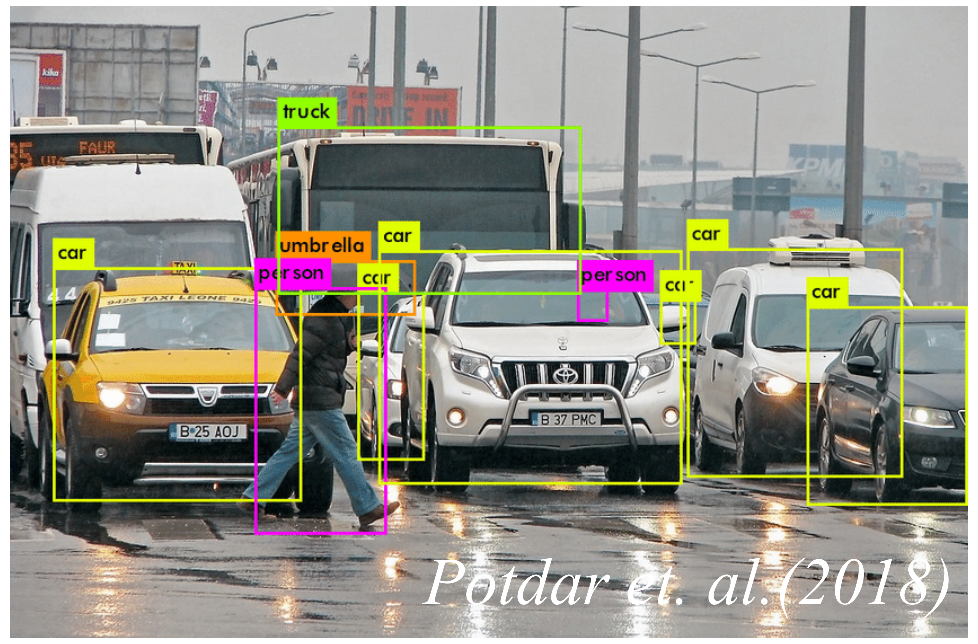

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The safe control problem in dynamic environments

Compute **control inputs** so that the system avoids **dynamic agents** with a probability of at least $1-\delta$.

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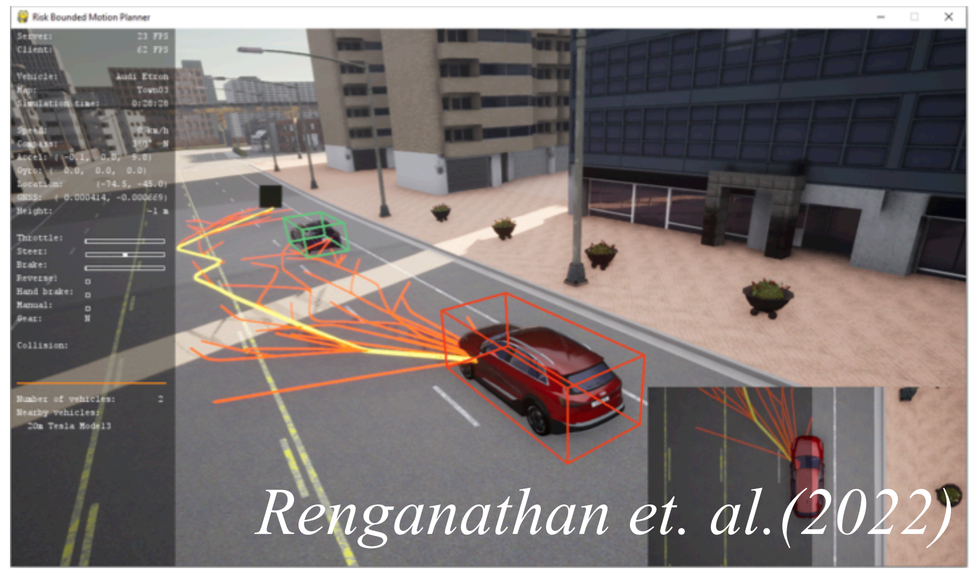
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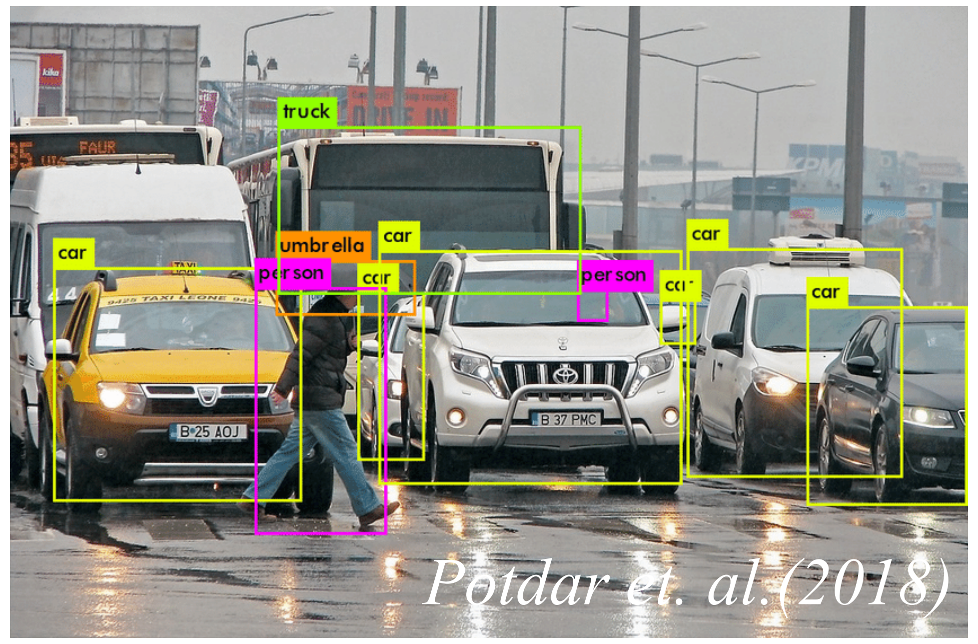
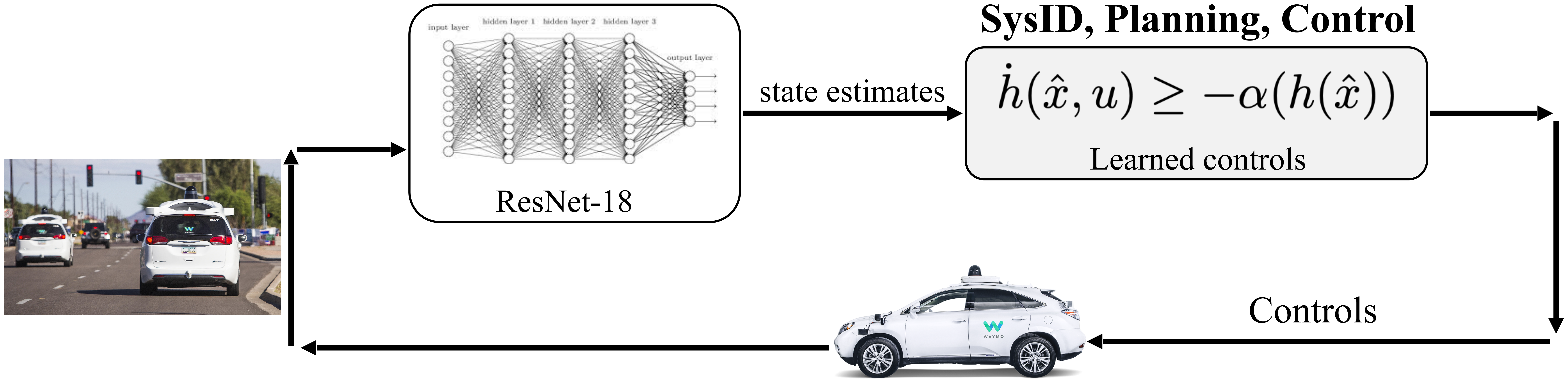


Potdar et. al.(2018)

How can we quantify uncertainty of **learning-enabled predictors** and design **safe control laws**?

Uncertainty quantification of learning-enabled systems



Sensing, Perception, Prediction

ResNet-18

state estimates

SysID, Planning, Control

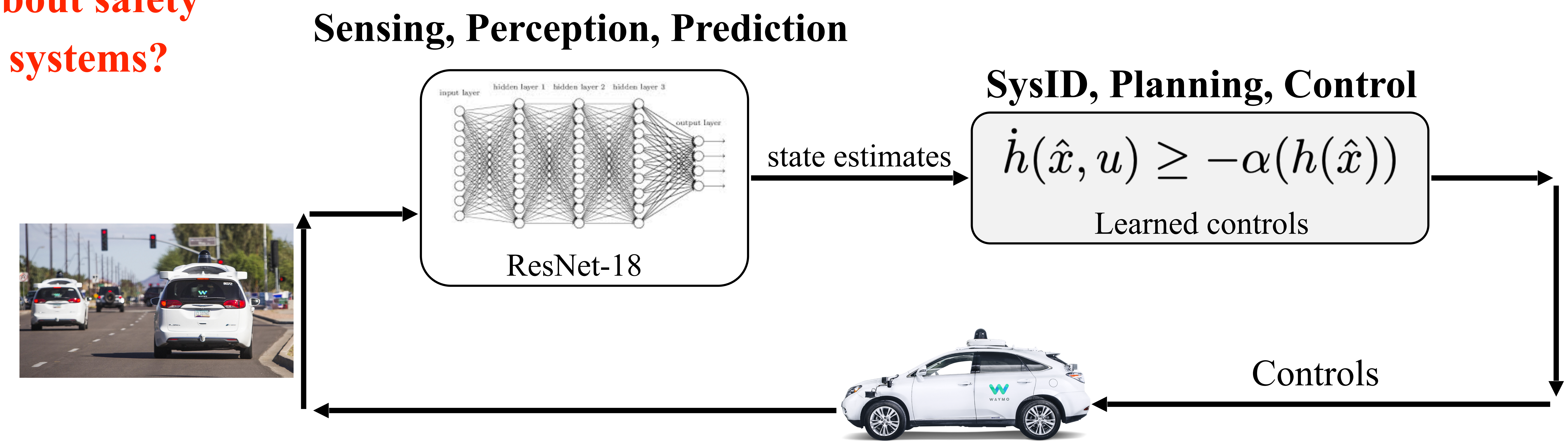
$$\dot{h}(\hat{x}, u) \geq -\alpha(h(\hat{x}))$$

Learned controls

Controls

Uncertainty quantification of learning-enabled systems

How do we reason about safety of high-dimensional systems?



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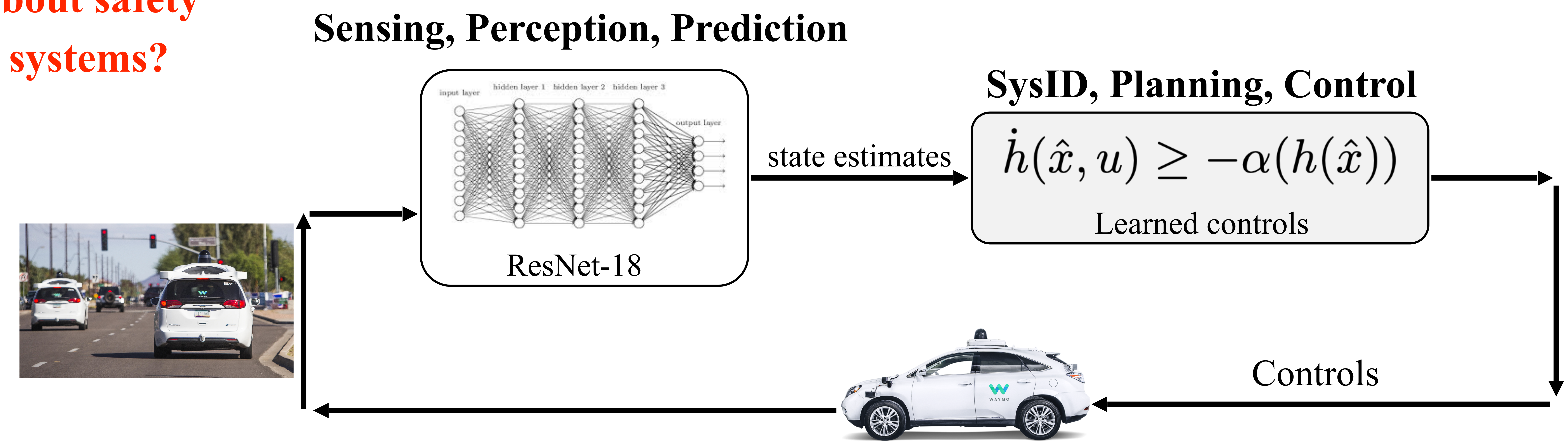

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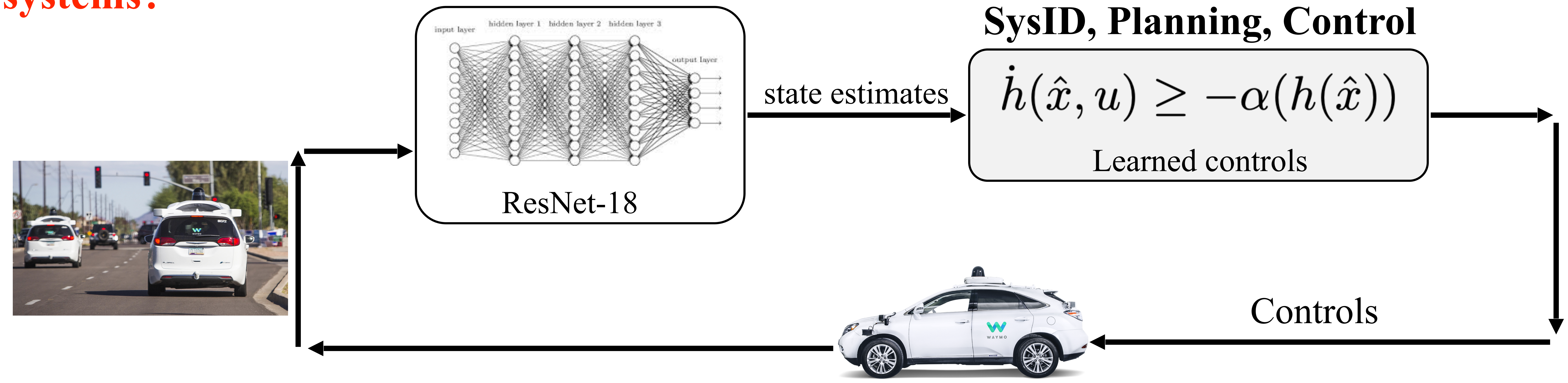

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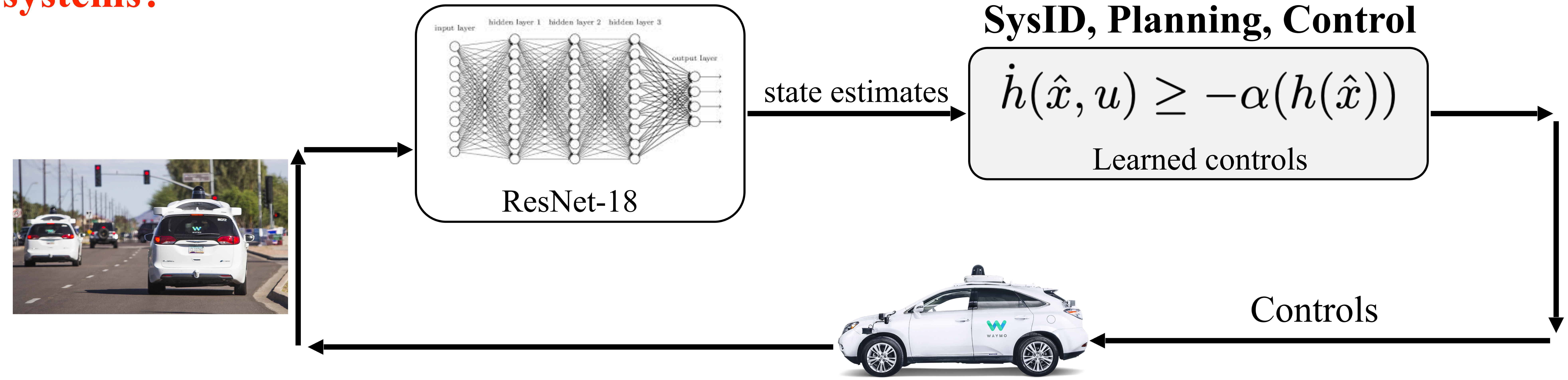

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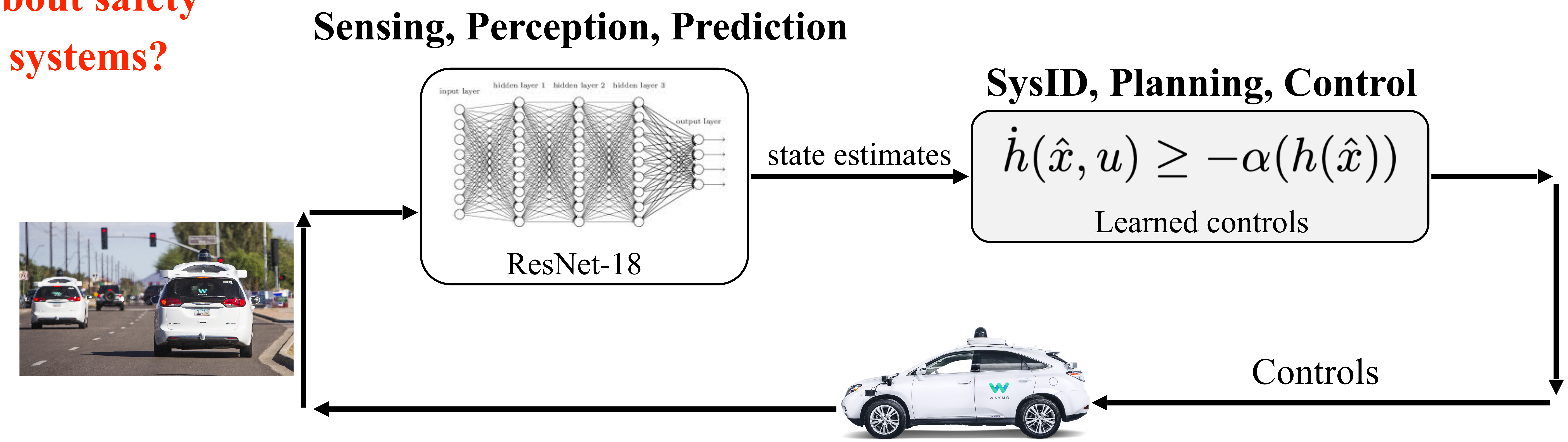

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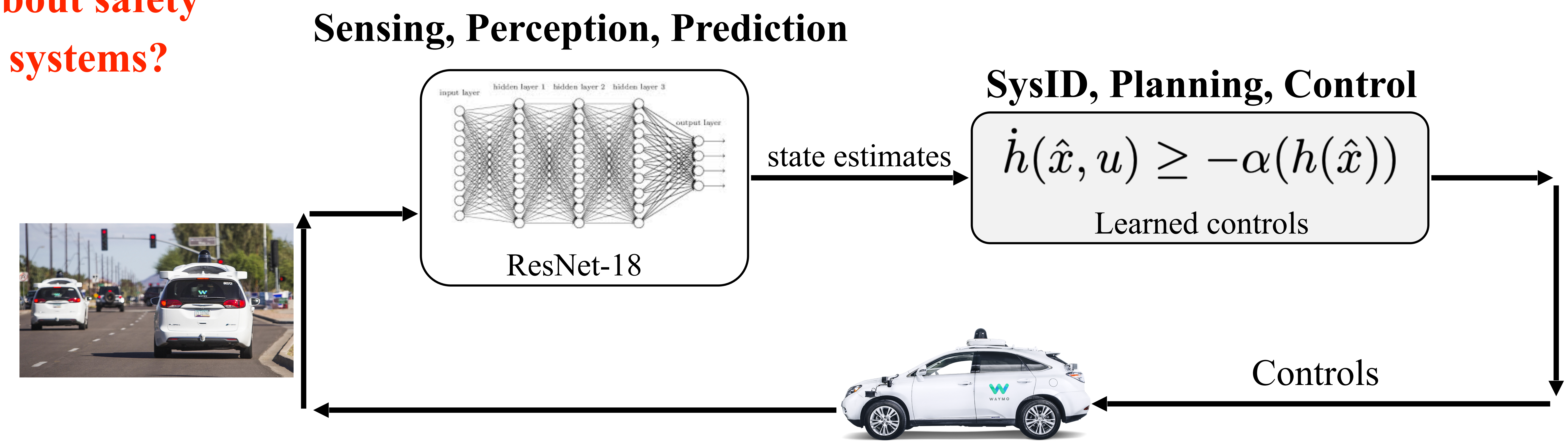

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Learned controls

Controls

Existing work (references omitted):

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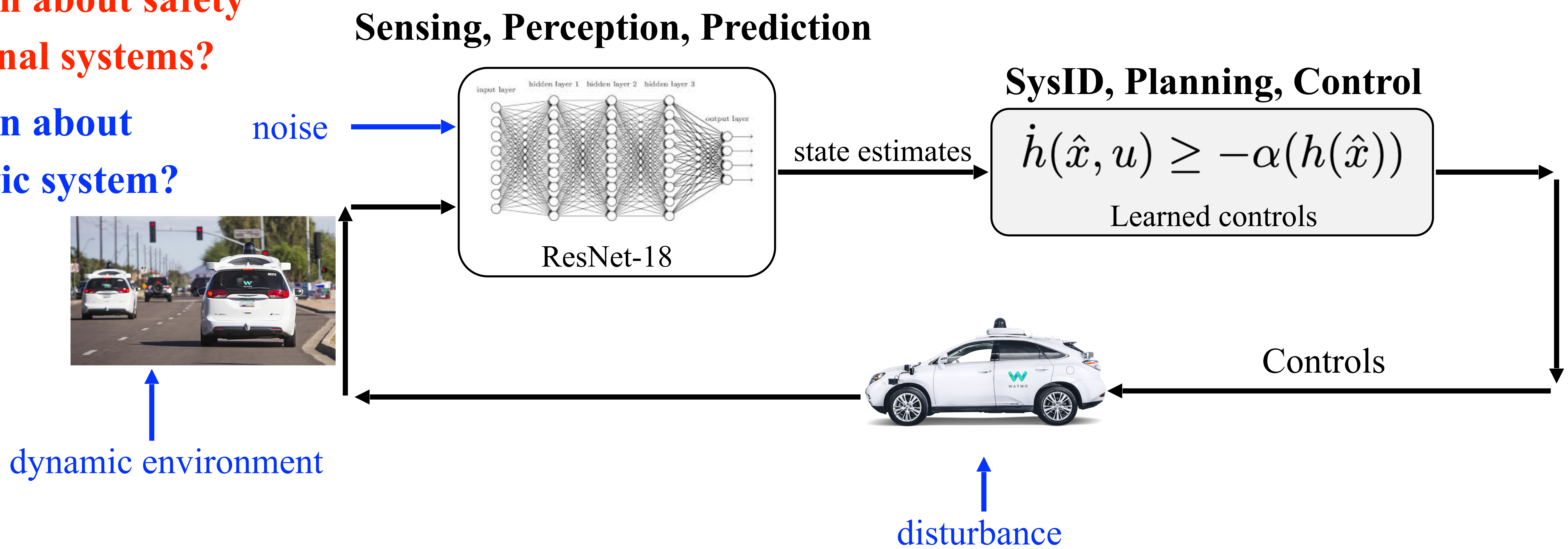
Uncertainty quantification of learning-enabled systems

How do we reason about safety of high-dimensional systems?

How do we reason about safety of stochastic system?



Sensing, Perception, Prediction

ResNet-18

noise

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Learned controls

Controls

dynamic environment

disturbance

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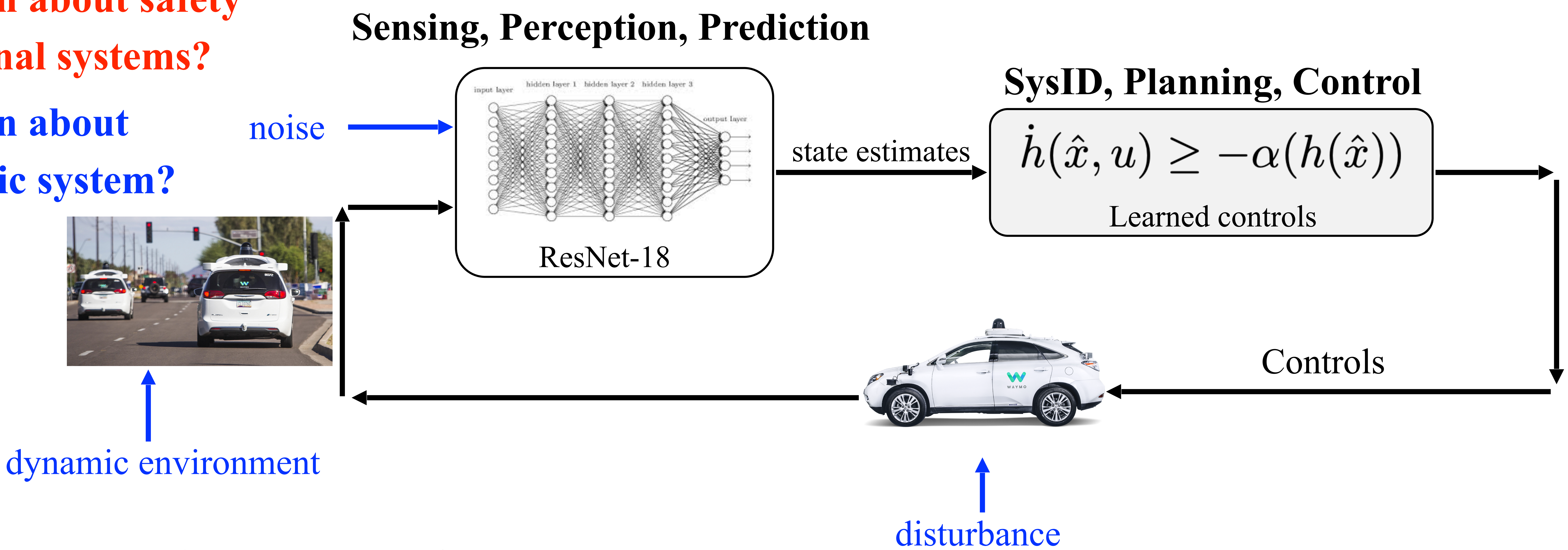

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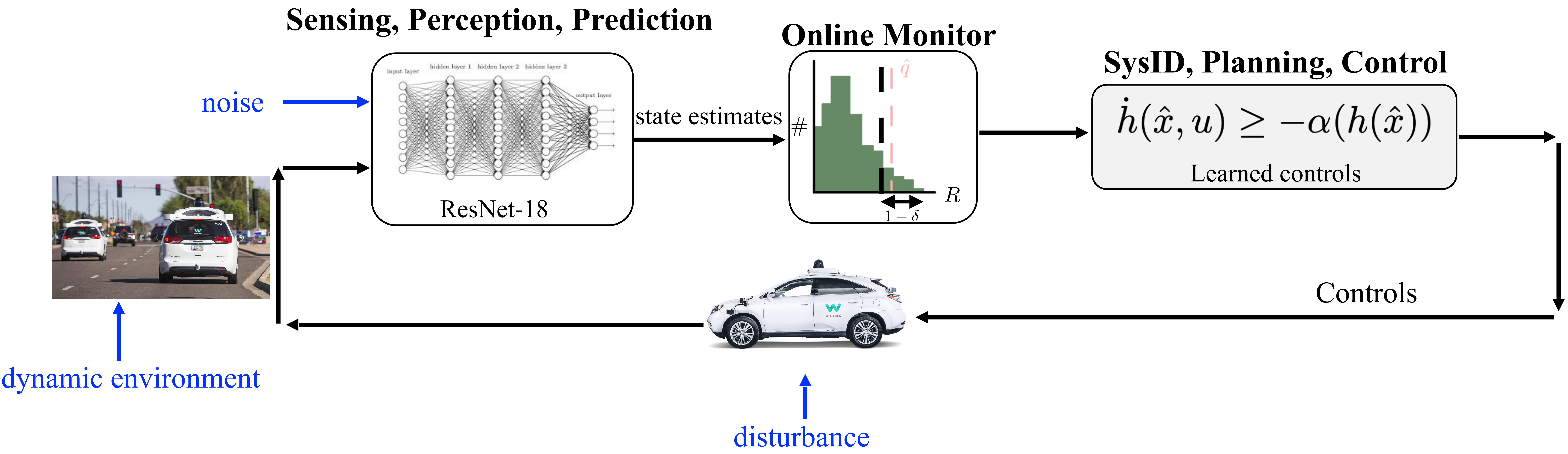
Can we **use statistical tools** to formally reason about safety of learning-enabled systems?

Uncertainty quantification of learning-enabled systems



Sensing, Perception, Prediction

Online Monitor

SysID, Planning, Control

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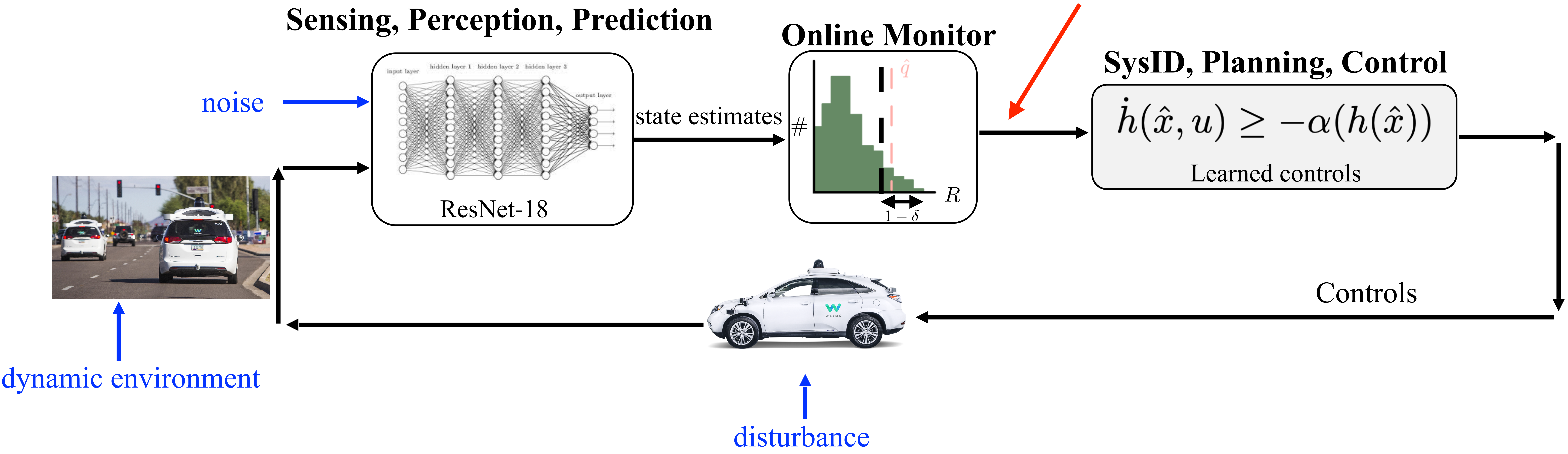
Uncertainty quantification of learning-enabled systems

uncertainty representation via efficient statistical abstractions



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Idea: Statistical uncertainty representations and safe control using **conformal prediction**.

Conformal Prediction

Conformal prediction in a nutshell

[1] Vovk and Shafer, "A Tutorial on Conformal Prediction", *JMLR,* 2008.

[2] Angelopoulos, "A Gentle Introduction to Conformal Prediction and Distribution-Free Uncertainty Quantification", *subm.,* 2021.

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Assumption: Availability of i.i.d. calibration data $\{(u^{(i)}, z^{(i)})\}_{i=1}^{k}$

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Goal[1]:

For a failure probability of $\delta \in (0, 1)$, we want to obtain a prediction region C s.t.

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Example[2]:

---

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Quantile lemma:

Let $R^{(0)}, R^{(1)}, \ldots, R^{(k)}$ be $k+1$ i.i.d. random variables.

Conformal prediction in a nutshell

nonconformity score (e.g., prediction error)

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test data

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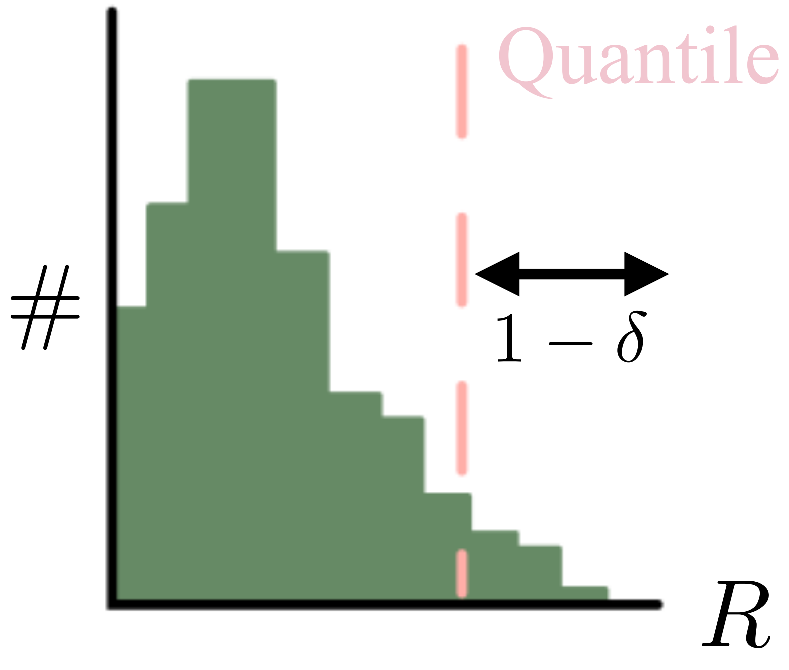

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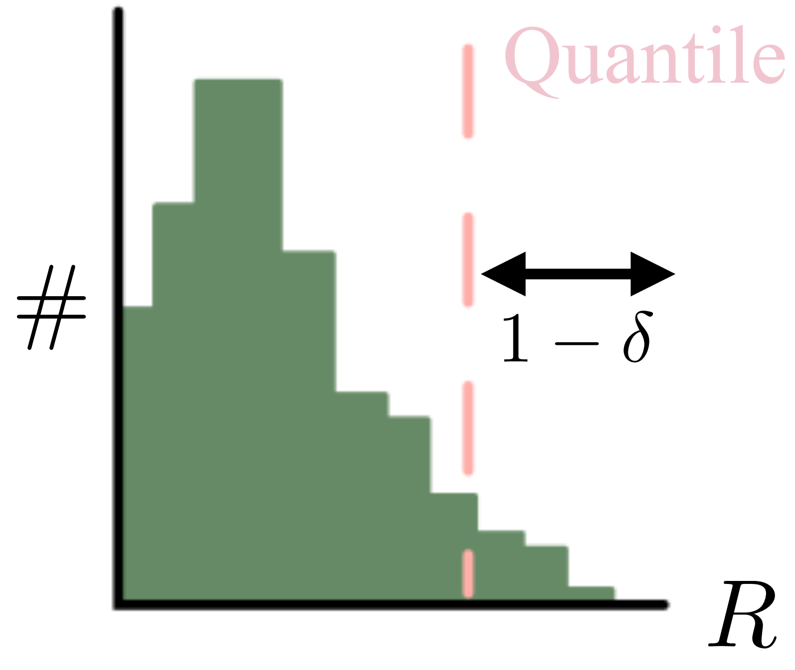
The $p := \lceil (k+1)(1-\delta) \rceil$ -th smallest nonconformity score



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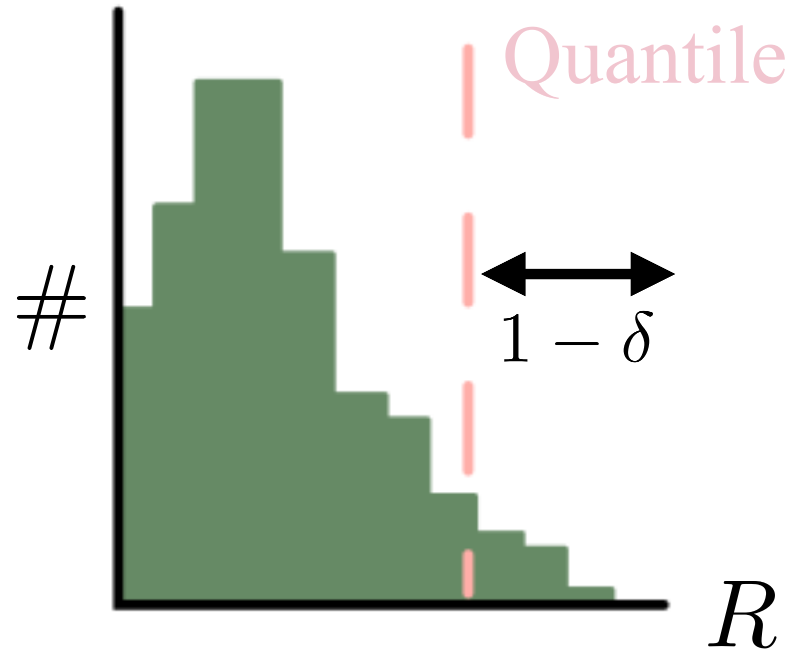

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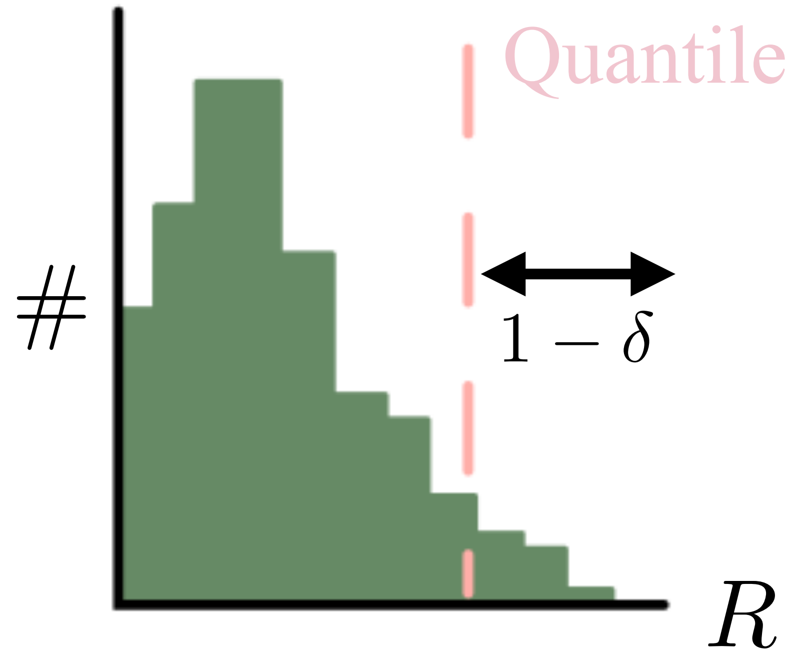

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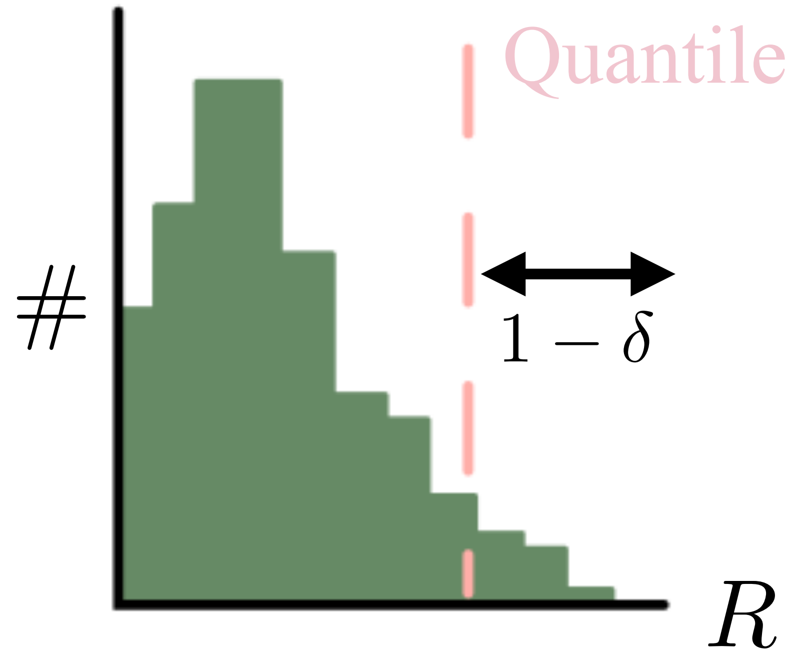

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$$\text{Prob}(z \in [\mu(u) - \text{Quantile}_{1-\delta}, \mu(u) + \text{Quantile}_{1-\delta}]) \geq 1 - \delta$$

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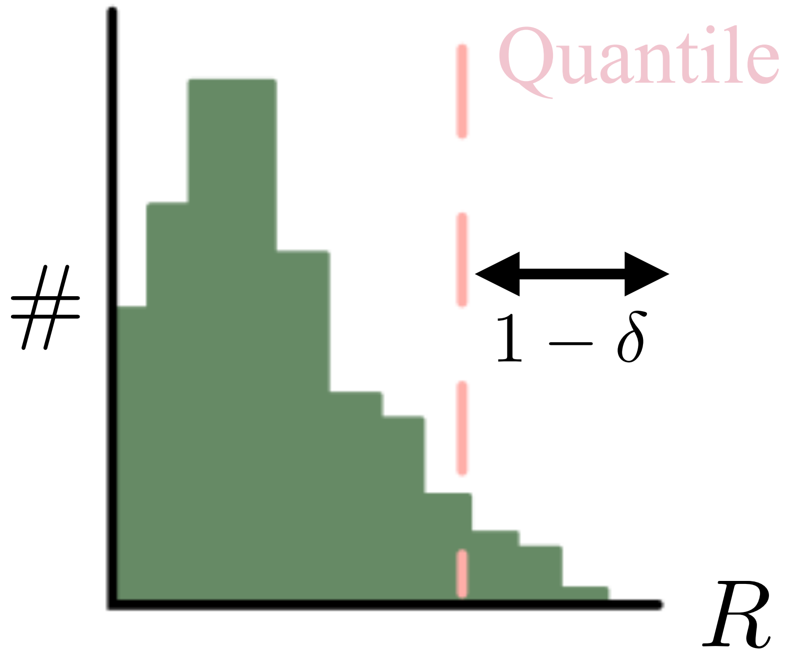

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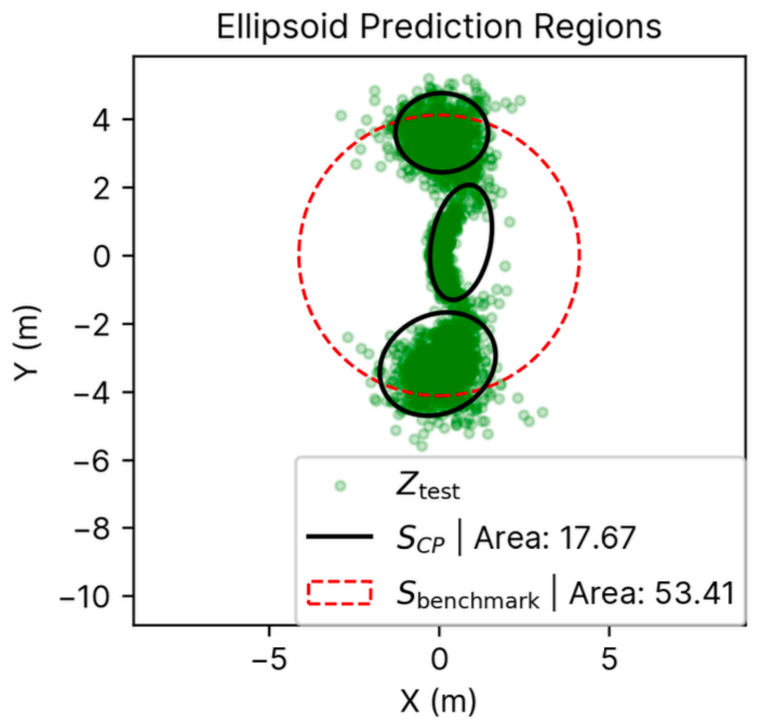
- **Simple, general, and efficient** with the right nonconformity score[1]:



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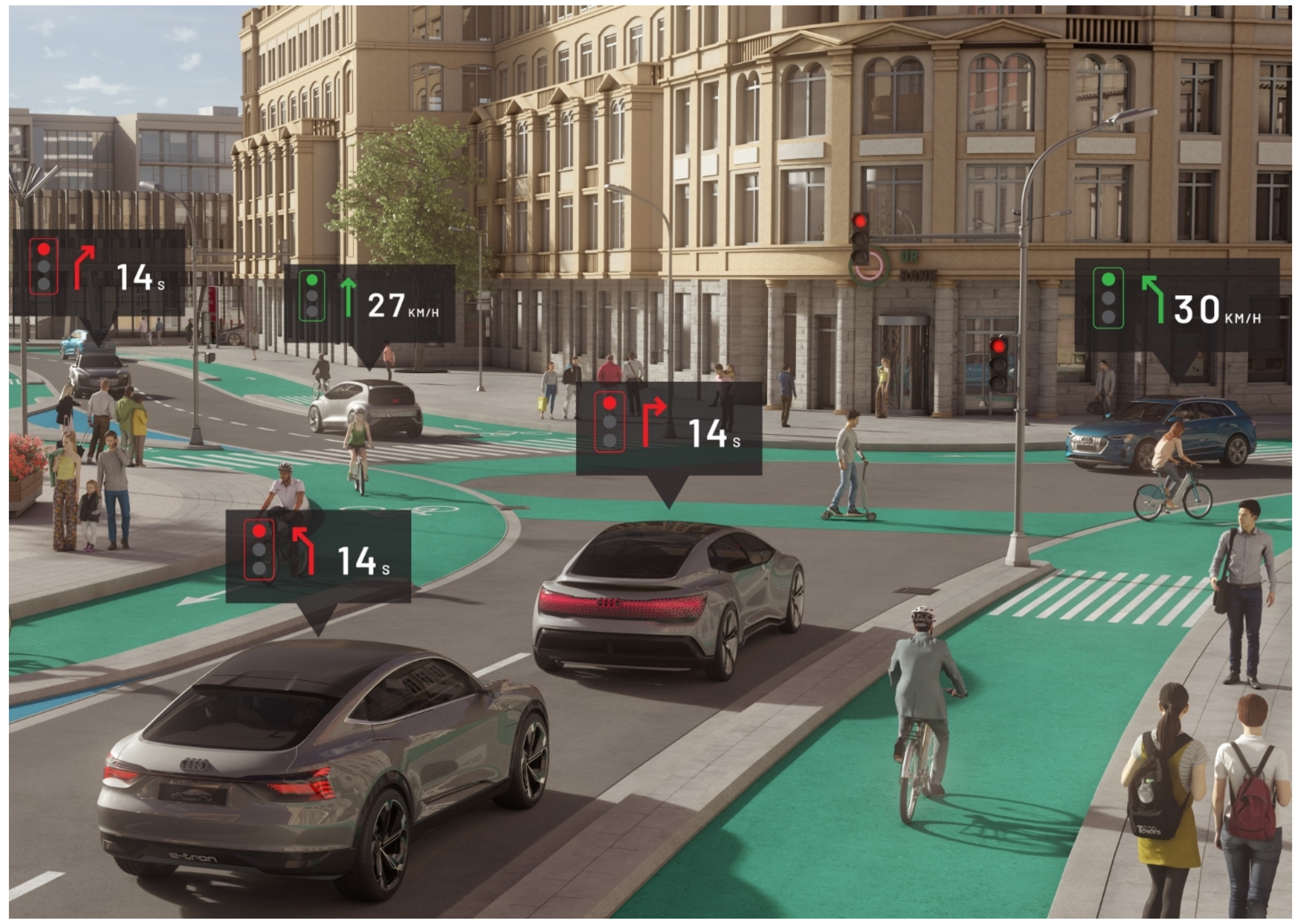
[1] Tumu, Cleaveland, Mangharam, Pappas, and **Lindemann**, “Multi-Modal Conformal Prediction Regions by Optimizing Convex Shape Templates”, *L4DC*, 2024.

Safe Control in Dynamic Environments

Safe control using conformal prediction



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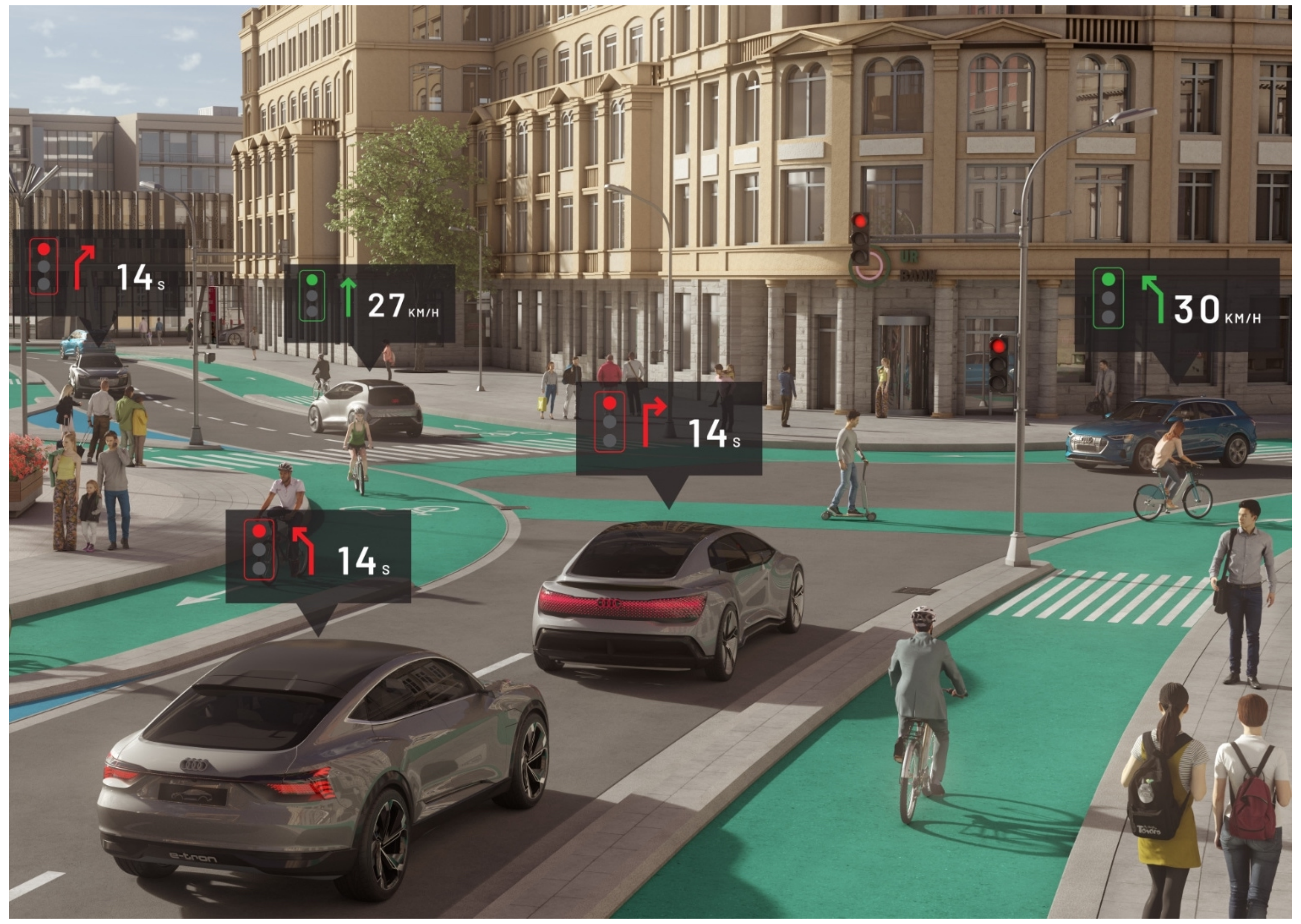

- **System Dynamics:** well understood and can be modeled accurately

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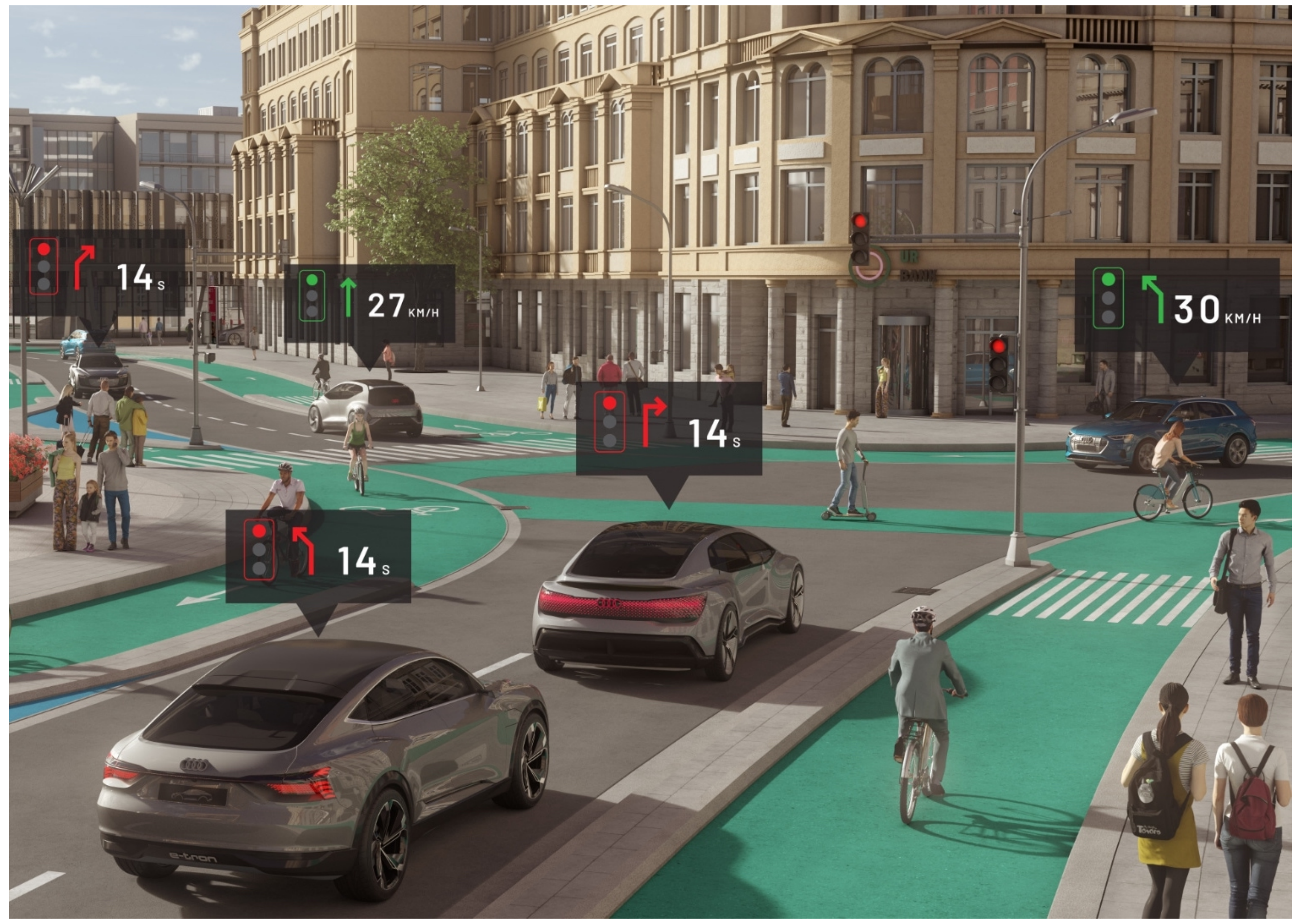


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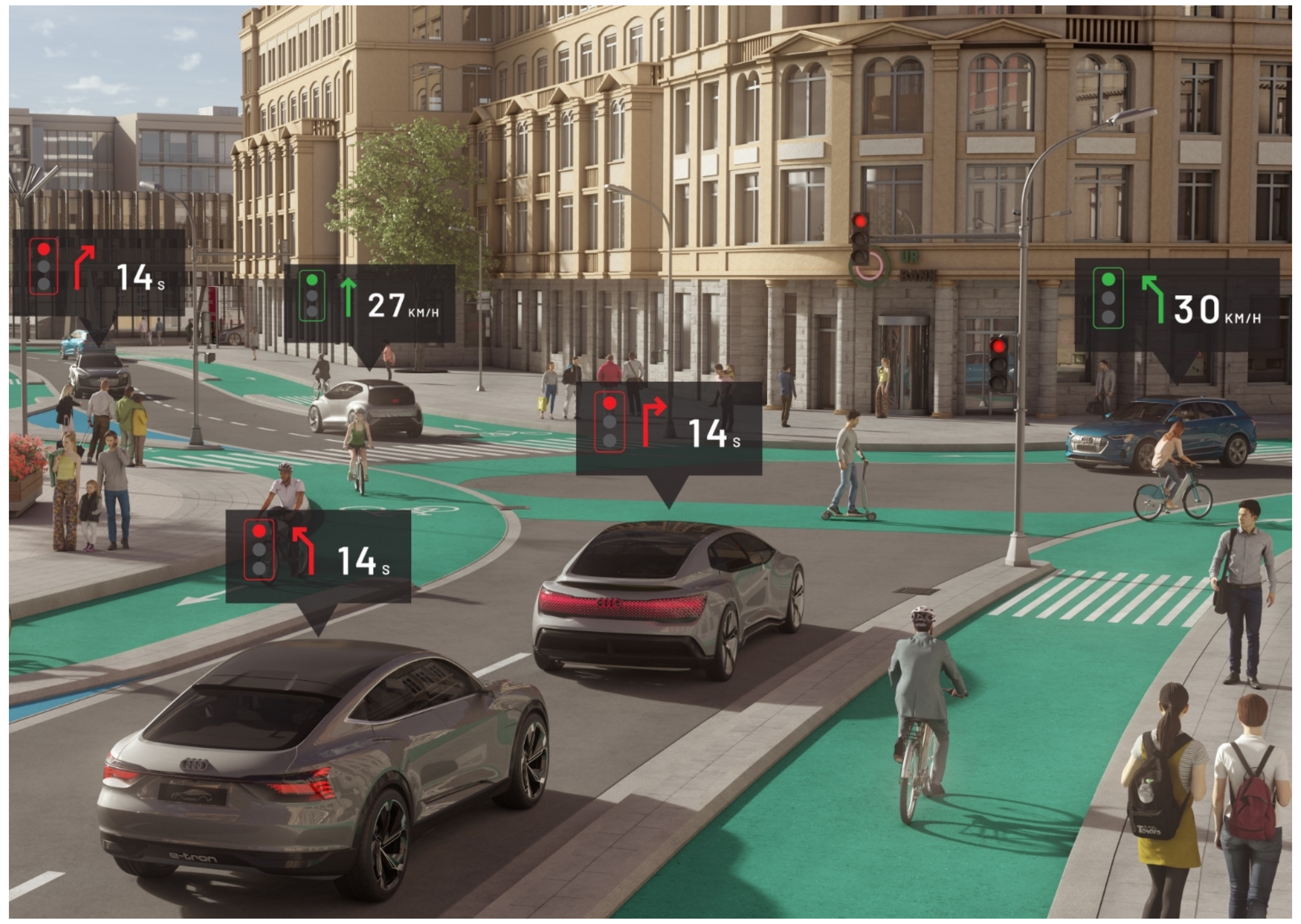

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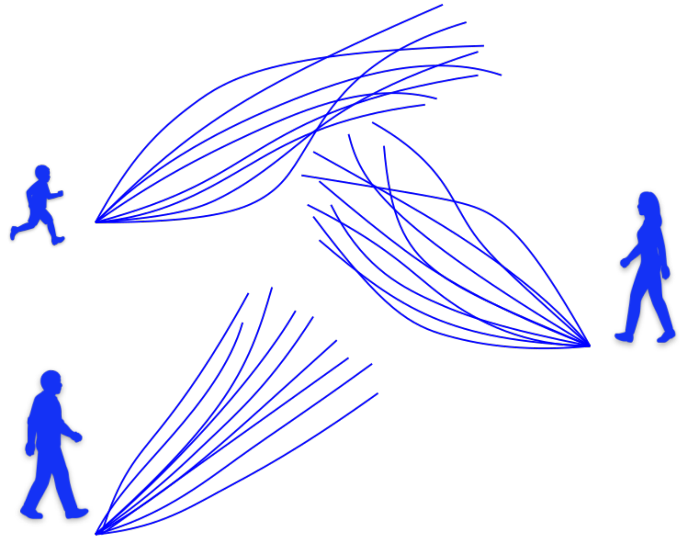
We use conformal prediction to design **safe and computationally efficient control** algorithms.

Overview

Offline trajectory dataset



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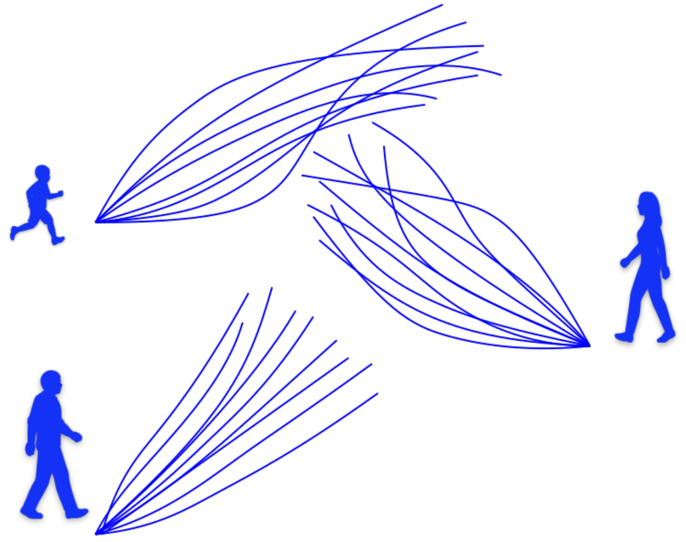
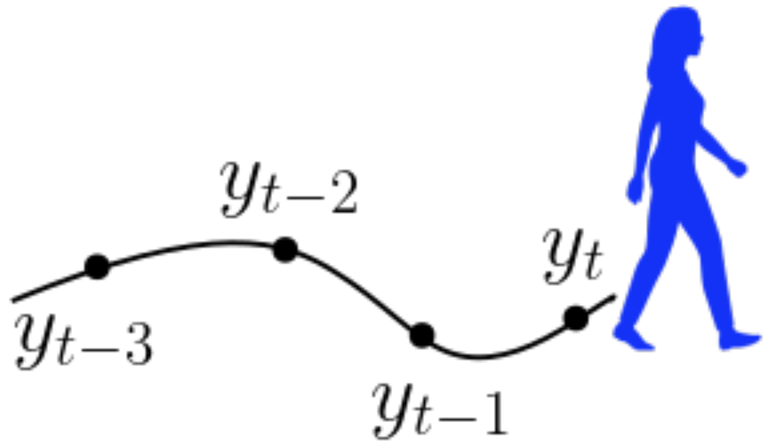


Online observations



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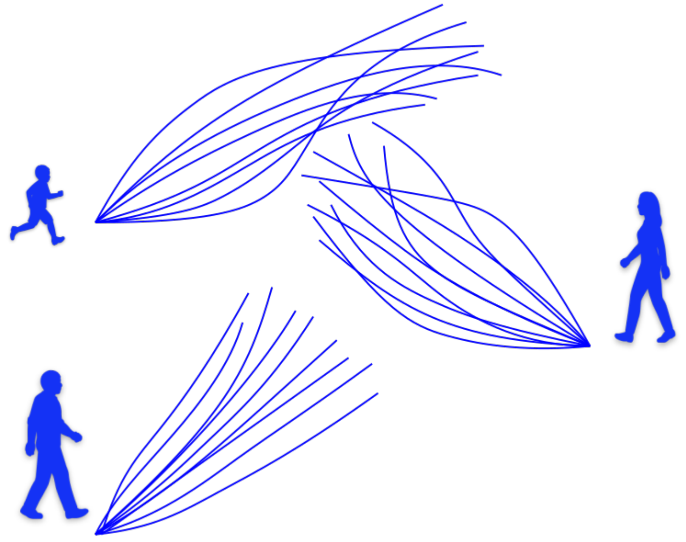

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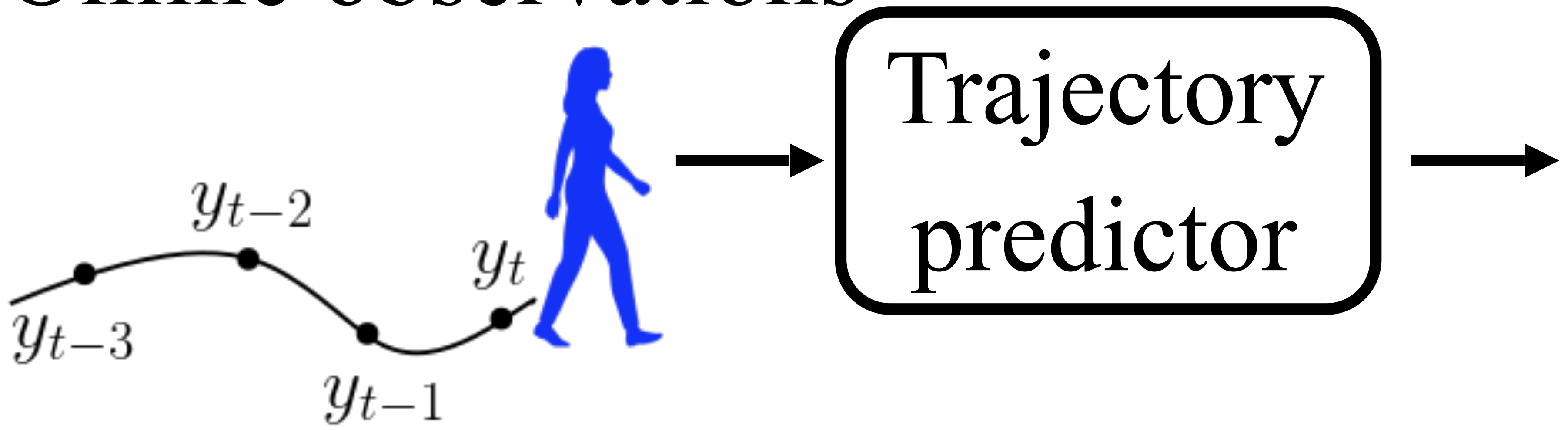

Trajectory predictor

Overview

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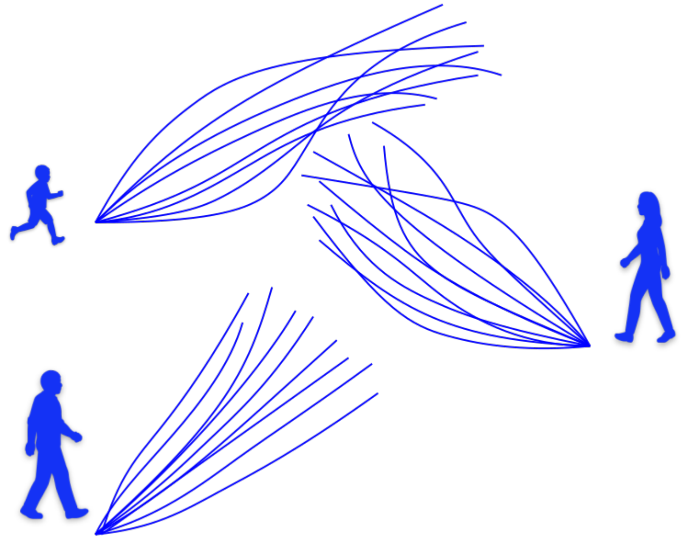

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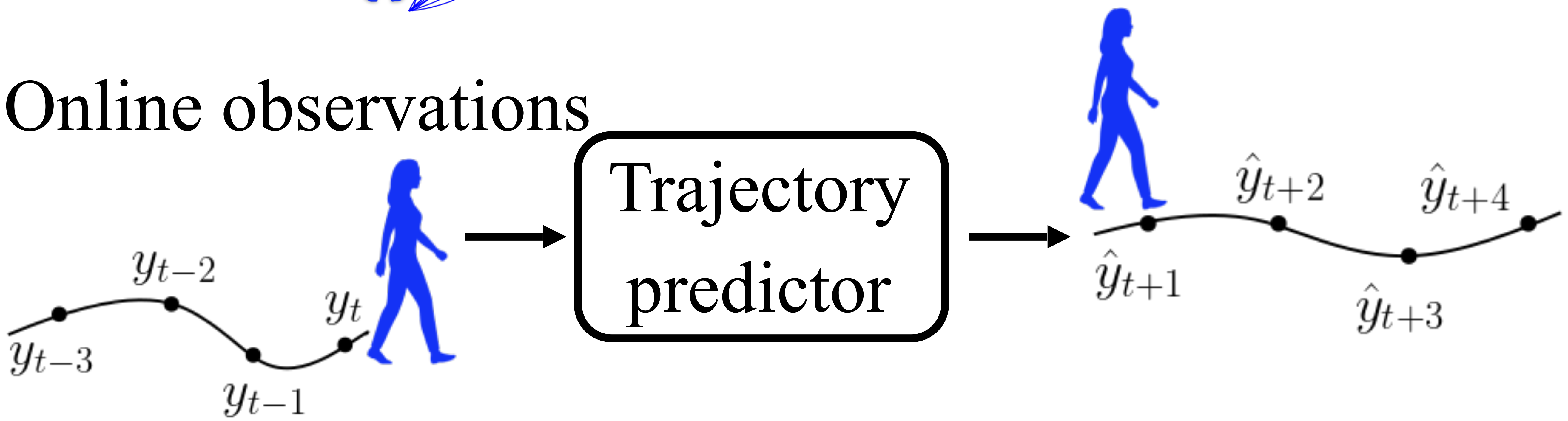

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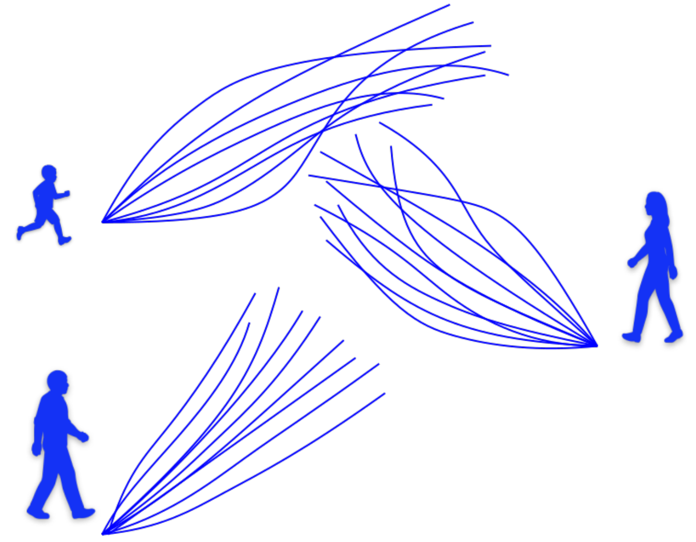

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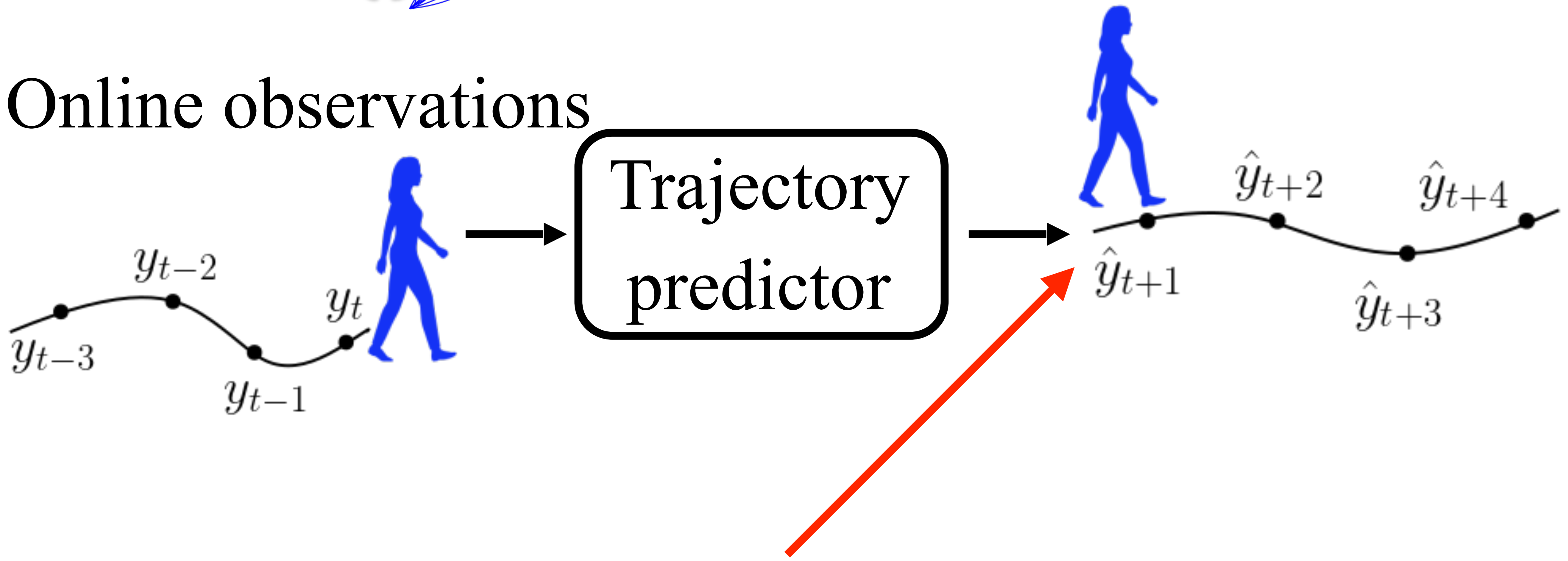
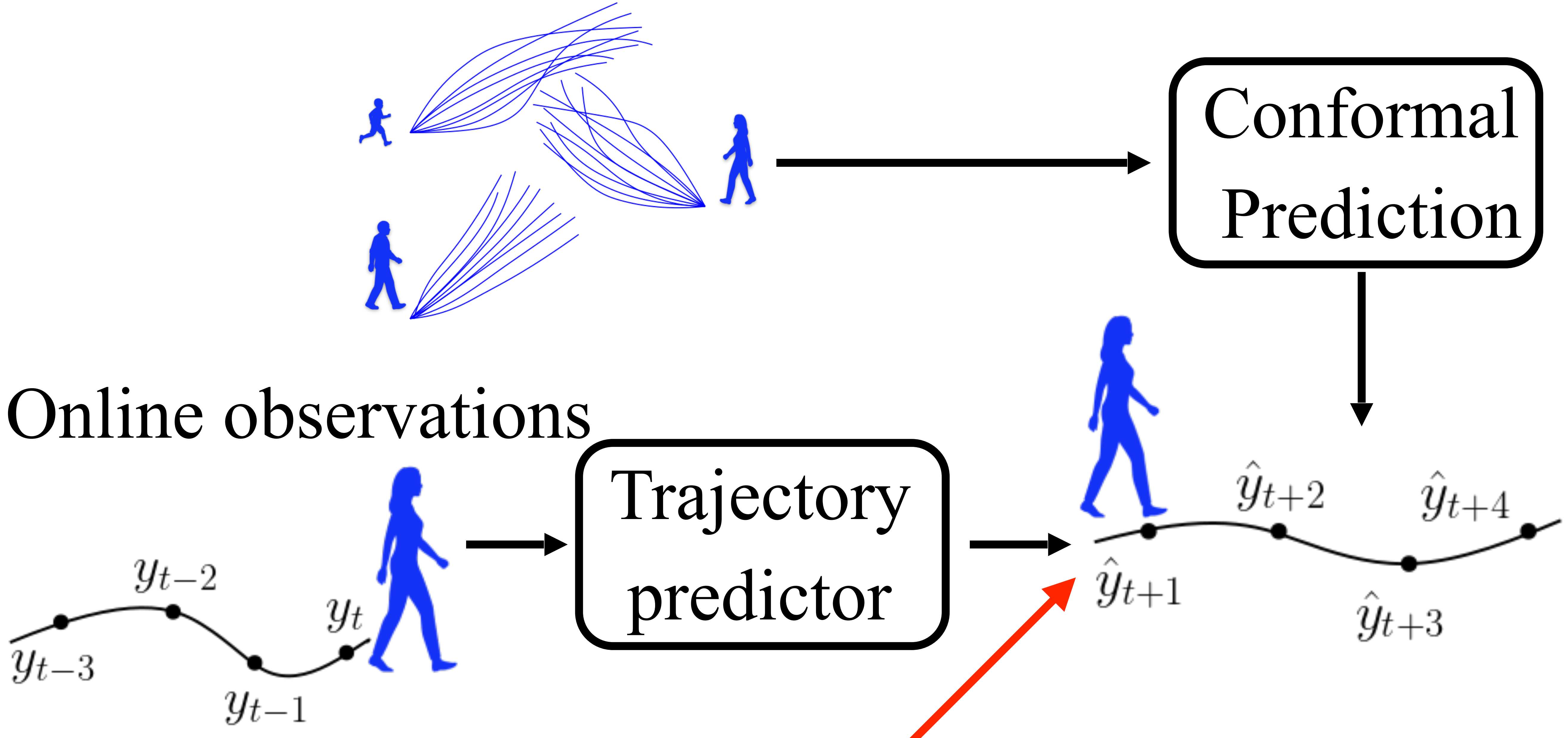


Trajectory predictor

How good are these estimates?

Overview



Offline trajectory dataset

Conformal Prediction

Online observations

Trajectory predictor

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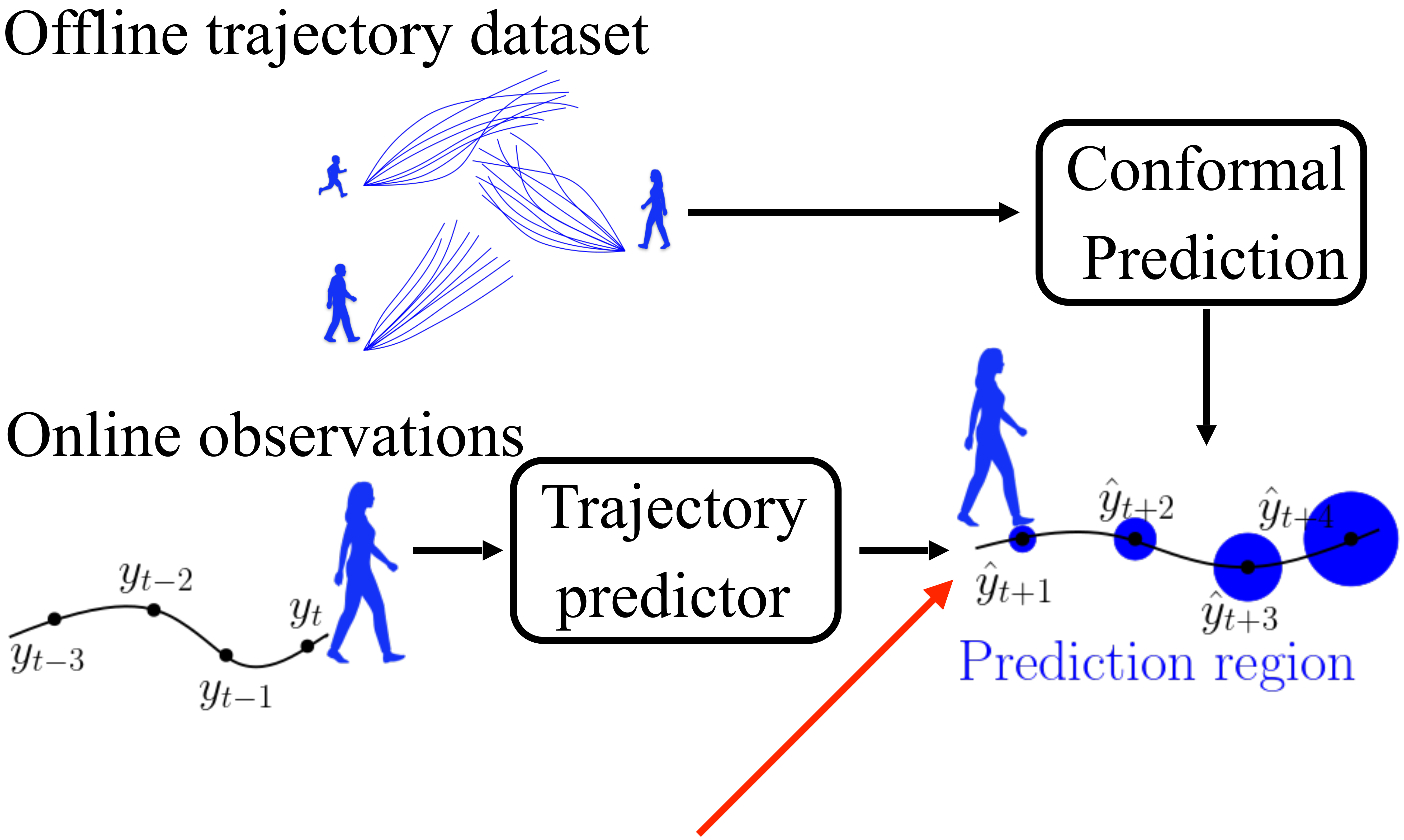

Offline trajectory dataset

Conformal Prediction

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Prediction region

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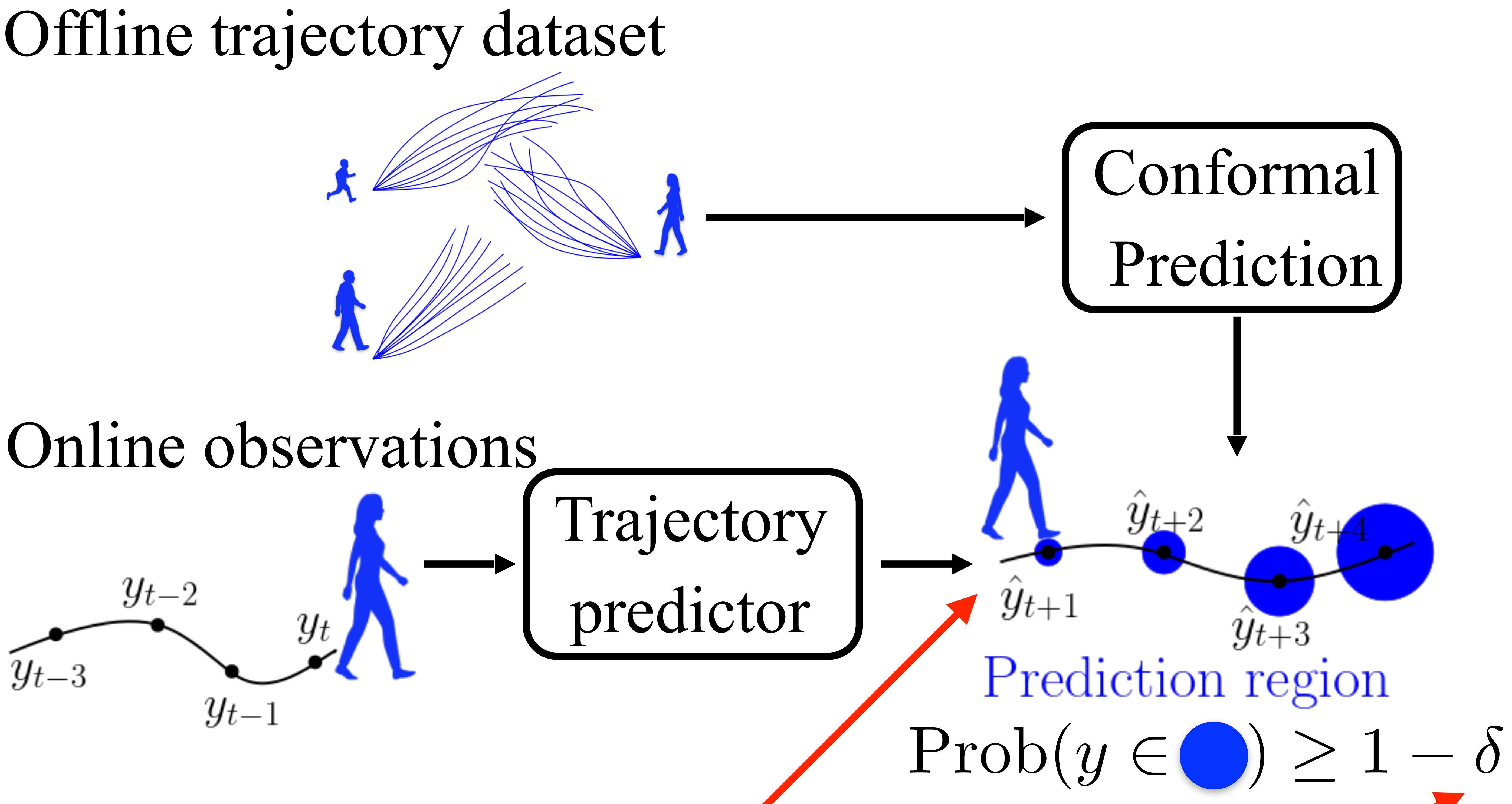


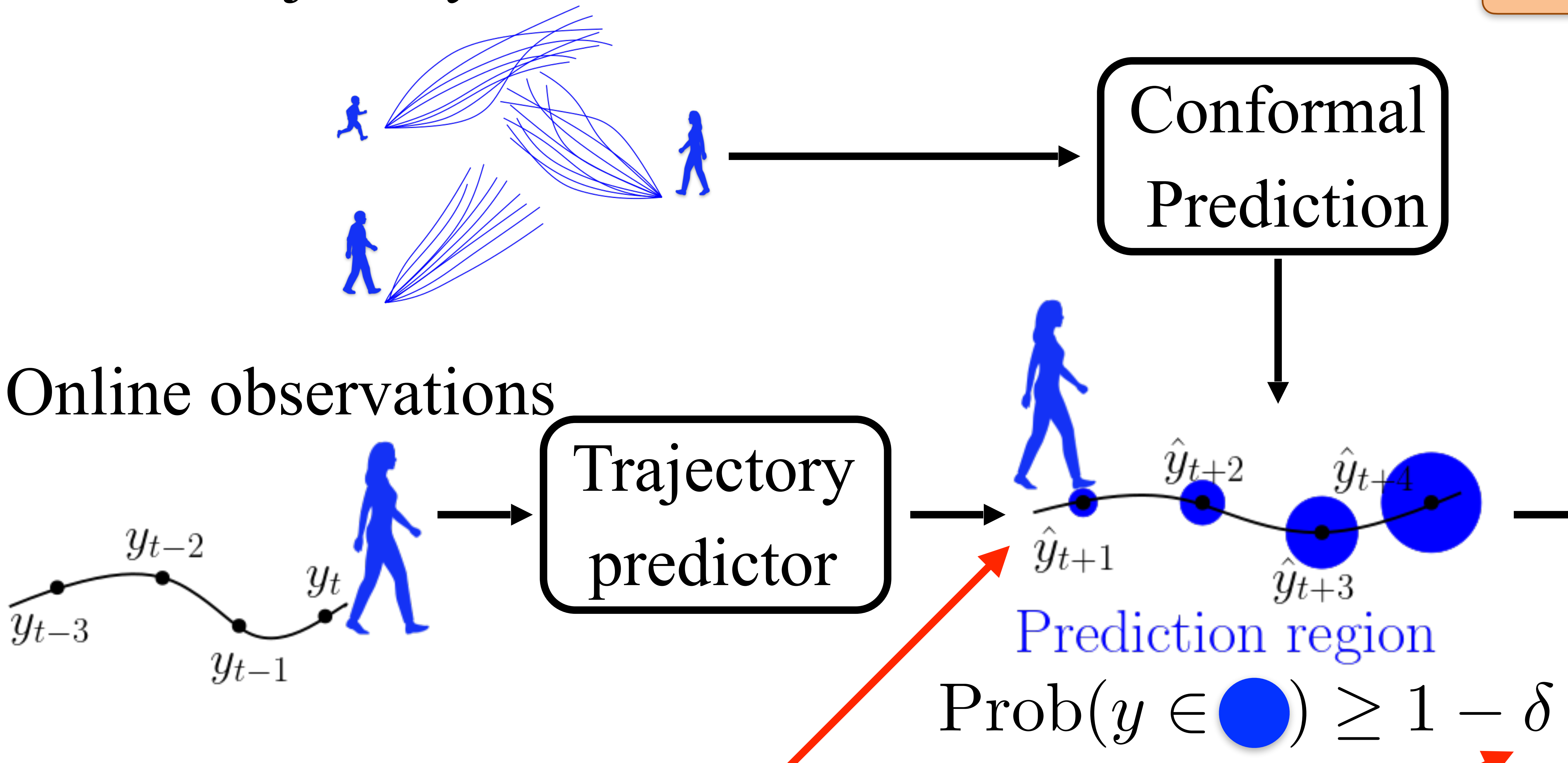
$\text{Prob}(y \in \bullet) \geq 1 - \delta$

How good are these estimates?

failure probability

Overview

Goal: Design a probabilistically safe controller.



Offline trajectory dataset

Conformal Prediction

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Dynamics

$x_{t+1} = f(x_t, u_t)$

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safety constraint

Model Predictive Control

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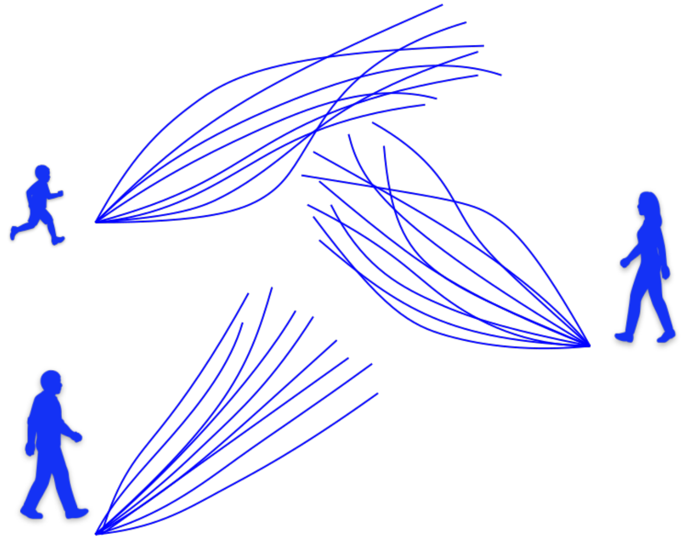

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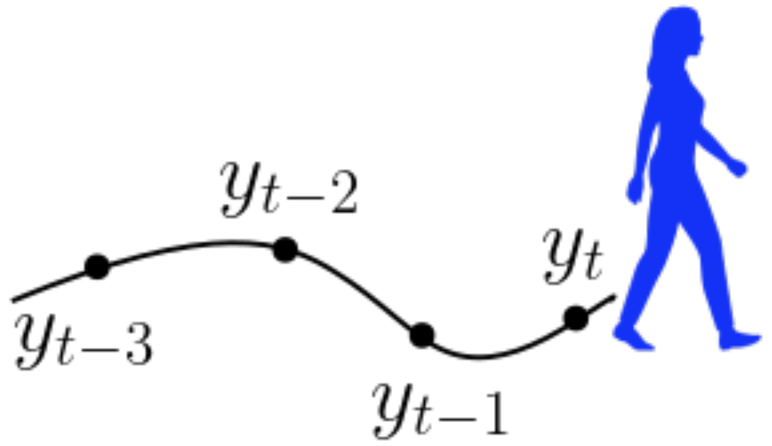

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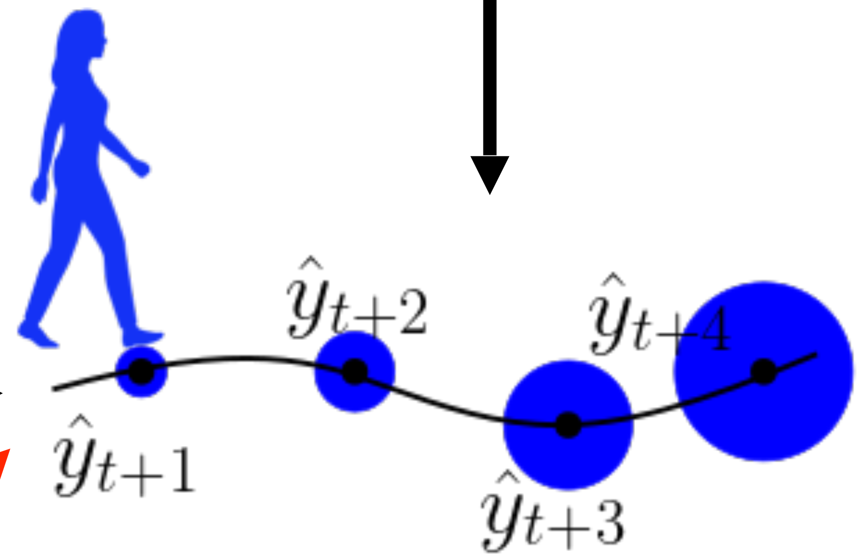

Prediction region

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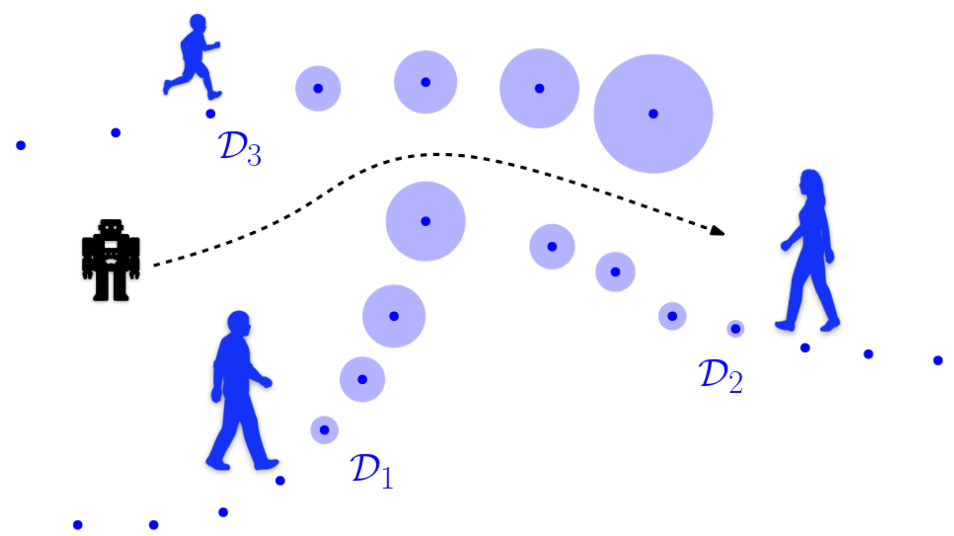
Contributions:

- Computationally lightweight algorithm
- Probabilistic safety guarantees
- Temporal logic constraints
- Deals with distribution shifts

Uncertainty representation in dynamic environments

Uncertainty representation in dynamic environments

- Let $(y_0, y_1, \ldots) \sim \mathcal{D}$ be a random trajectory



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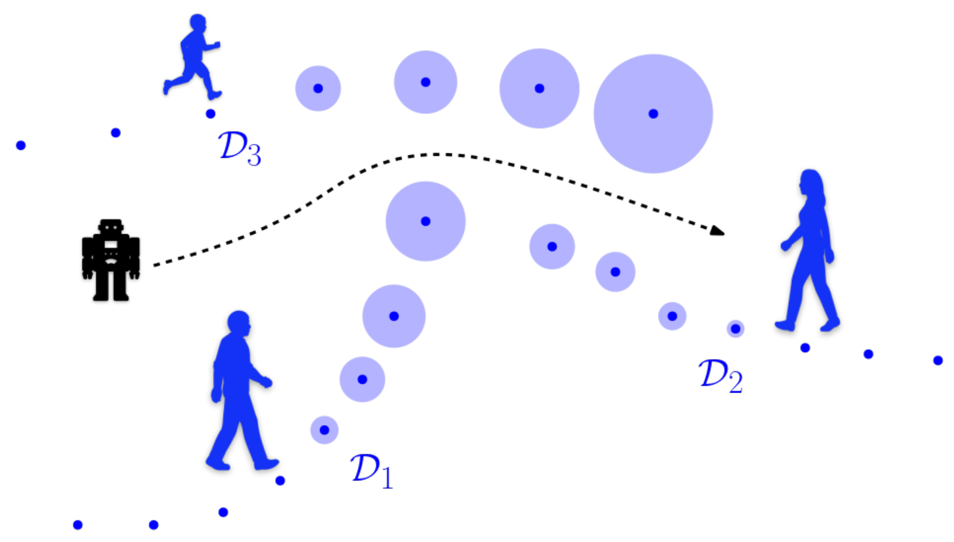

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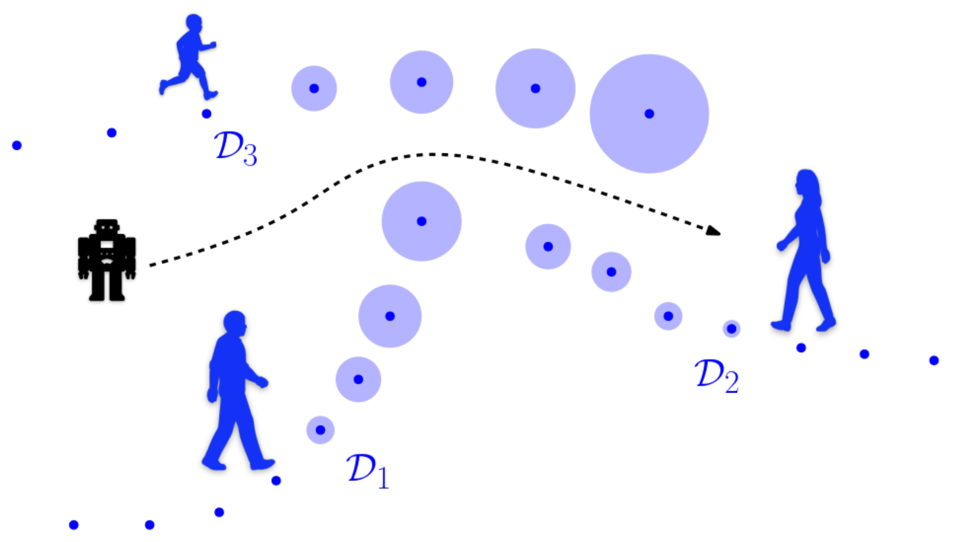

- Consider a trajectory predictor that maps $(y_0, \dots, y_t)$ to $(\hat{y}_{t+1}, \dots, \hat{y}_T)$

observations

predictions of $(y_{t+1}, \dots, y_T)$

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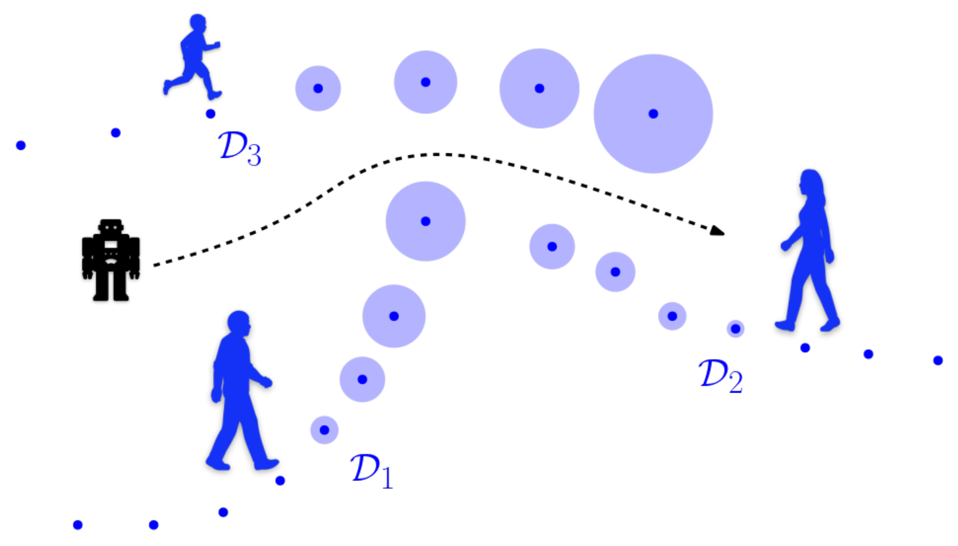

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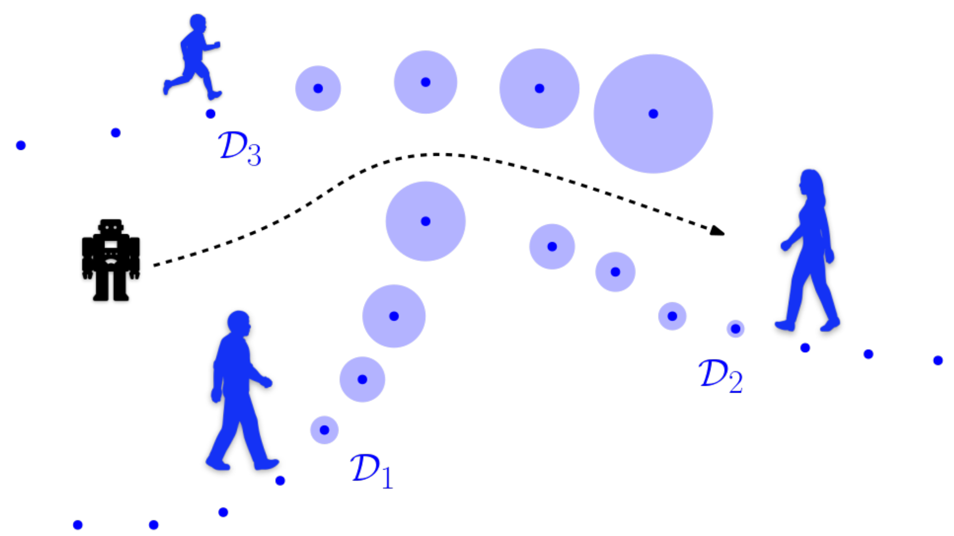

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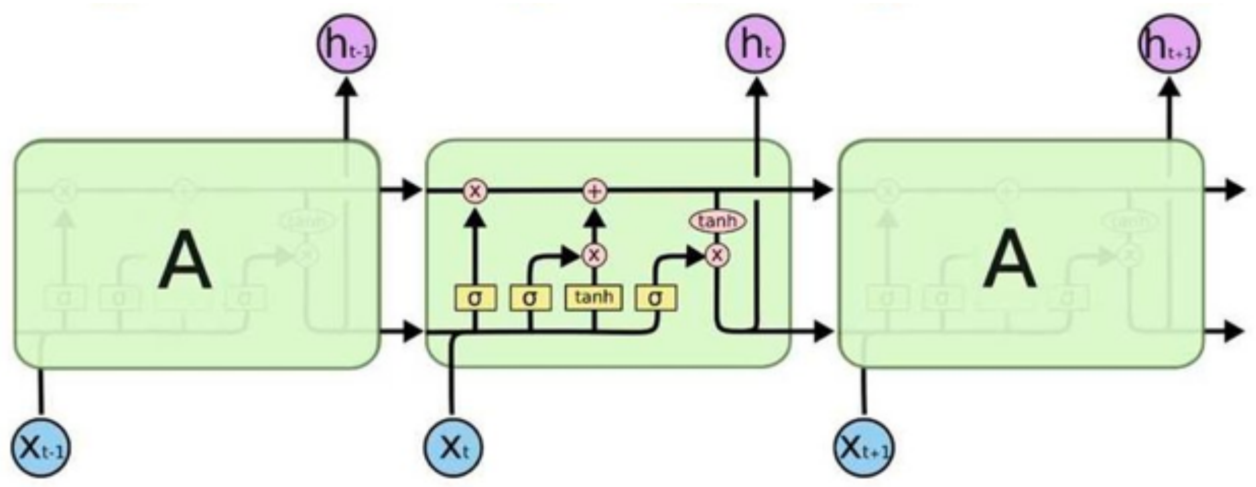
- For instance, social LSTMs



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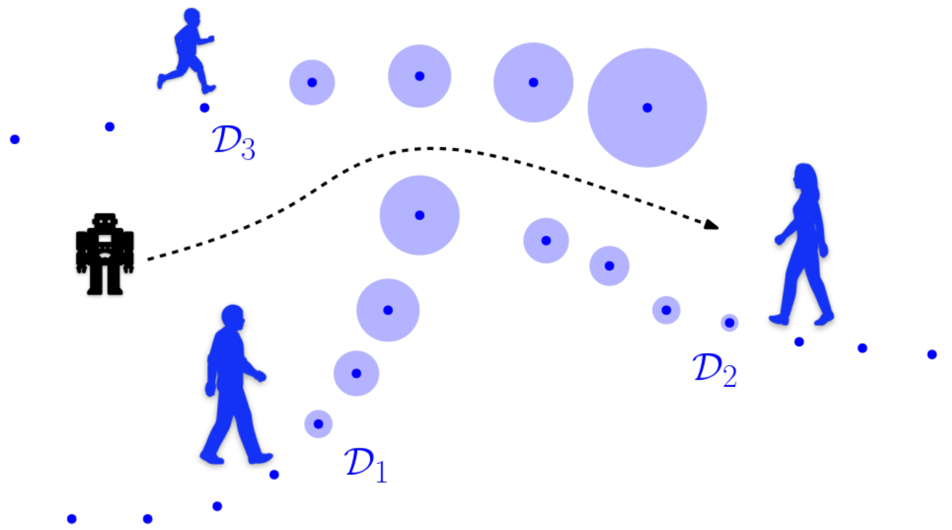

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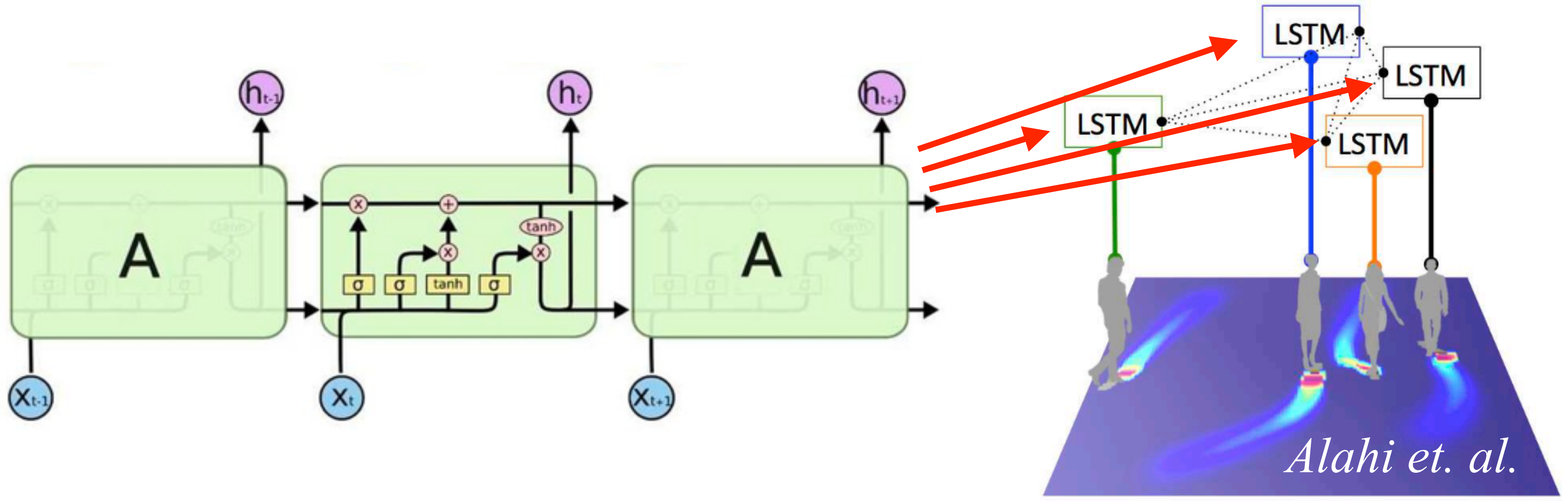

Alahi et. al.

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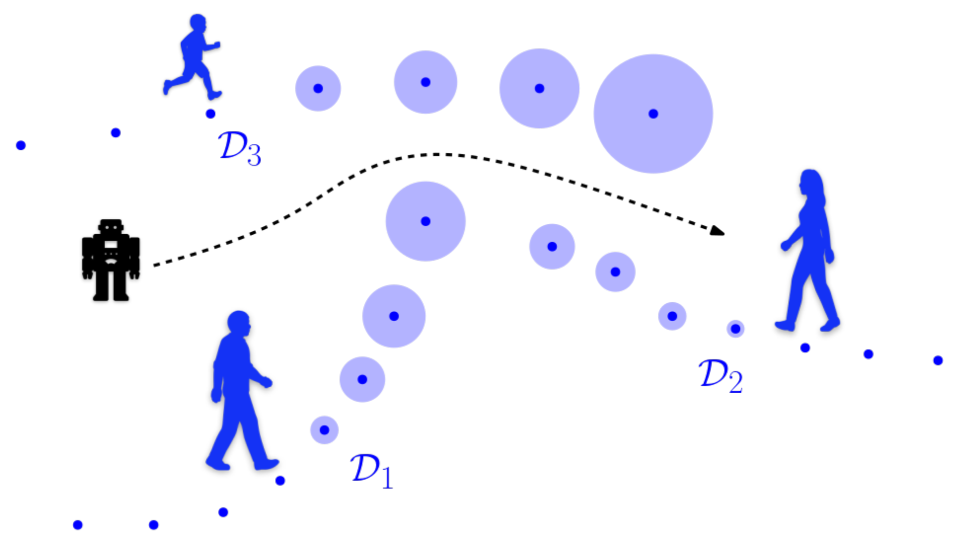

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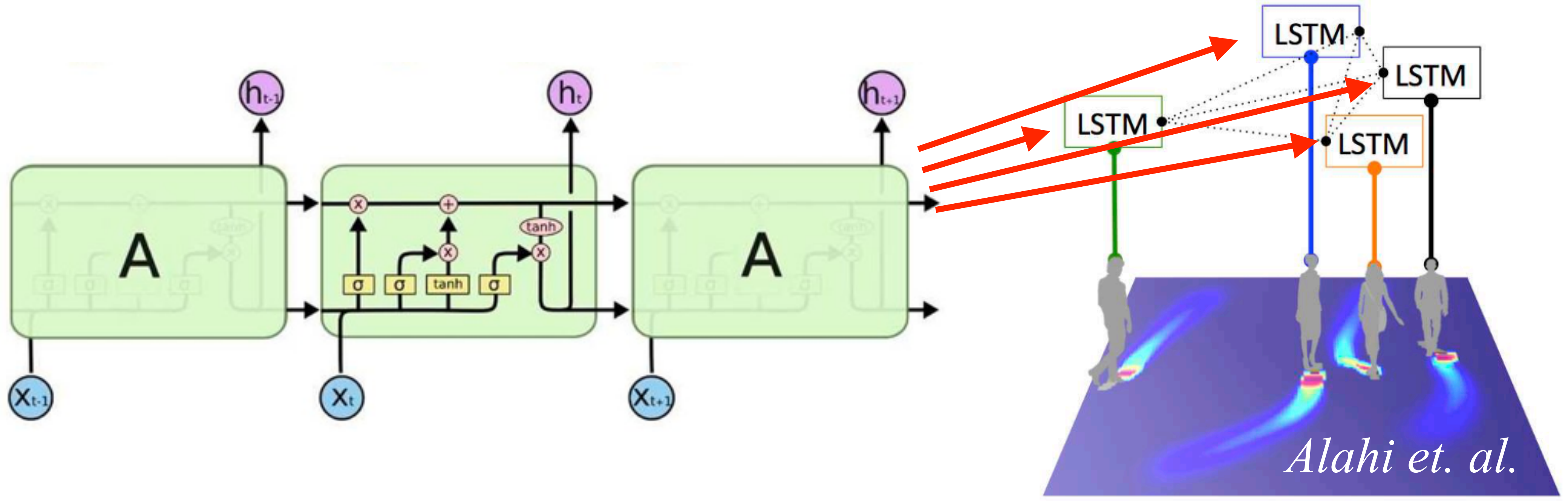
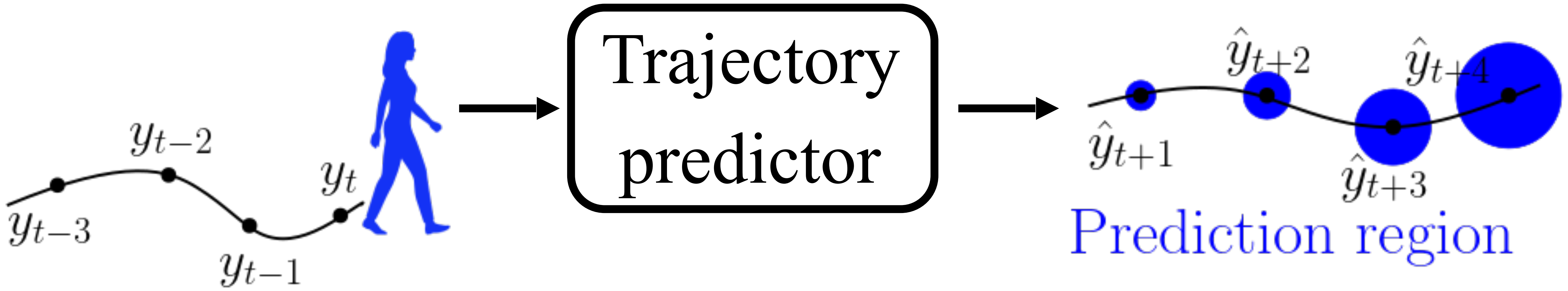


Alahi et. al.

- **Task**: quantify uncertainty at the state prediction level



Trajectory predictor

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Uncertainty representation in dynamic environments

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Example: Intersection with pedestrians in CARLA

Lindemann, Cleaveland, Shim, and Pappas, "Safe Planning in Dynamic Environments using Conformal Prediction", *RA-L,* 2023.

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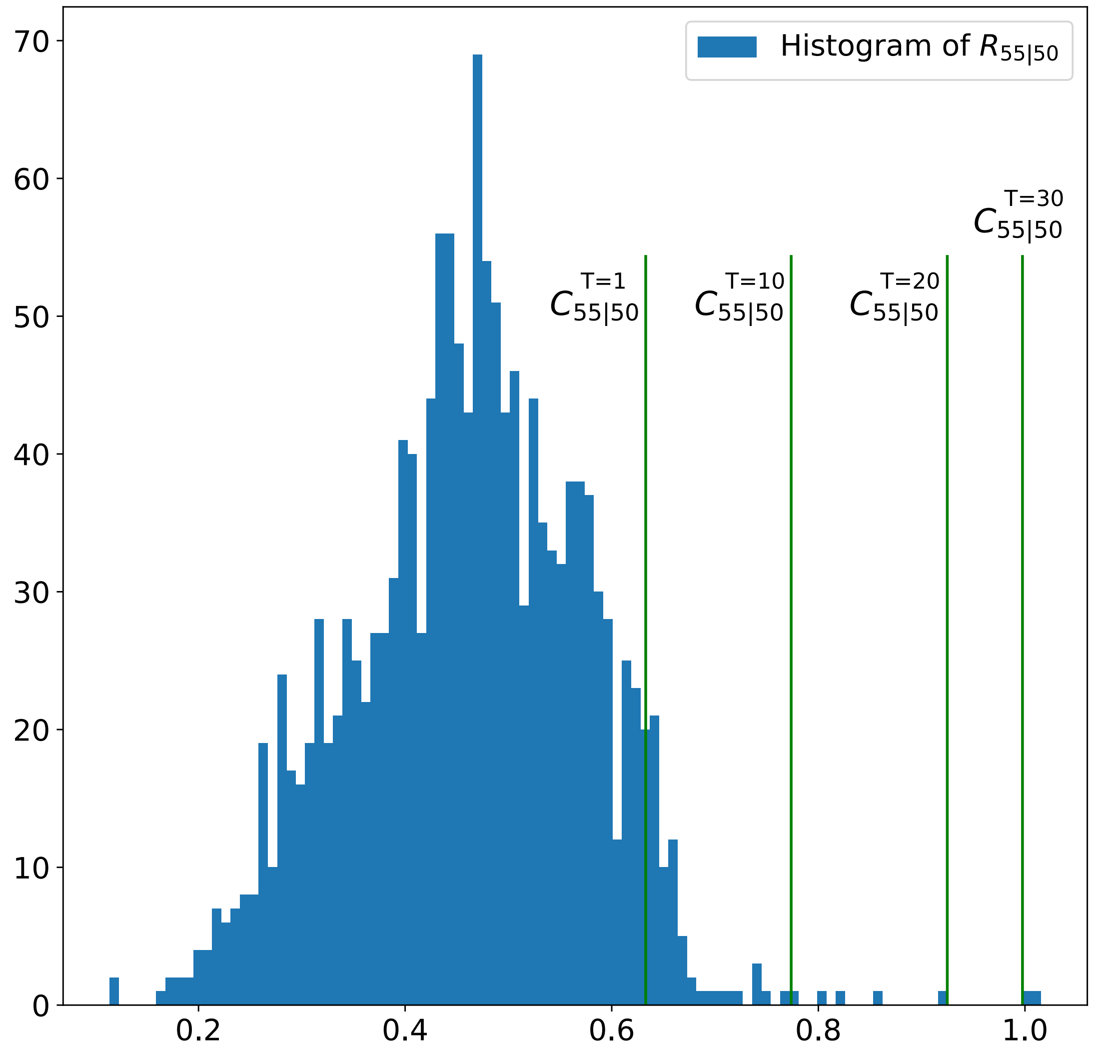

2.5 second ahead prediction error

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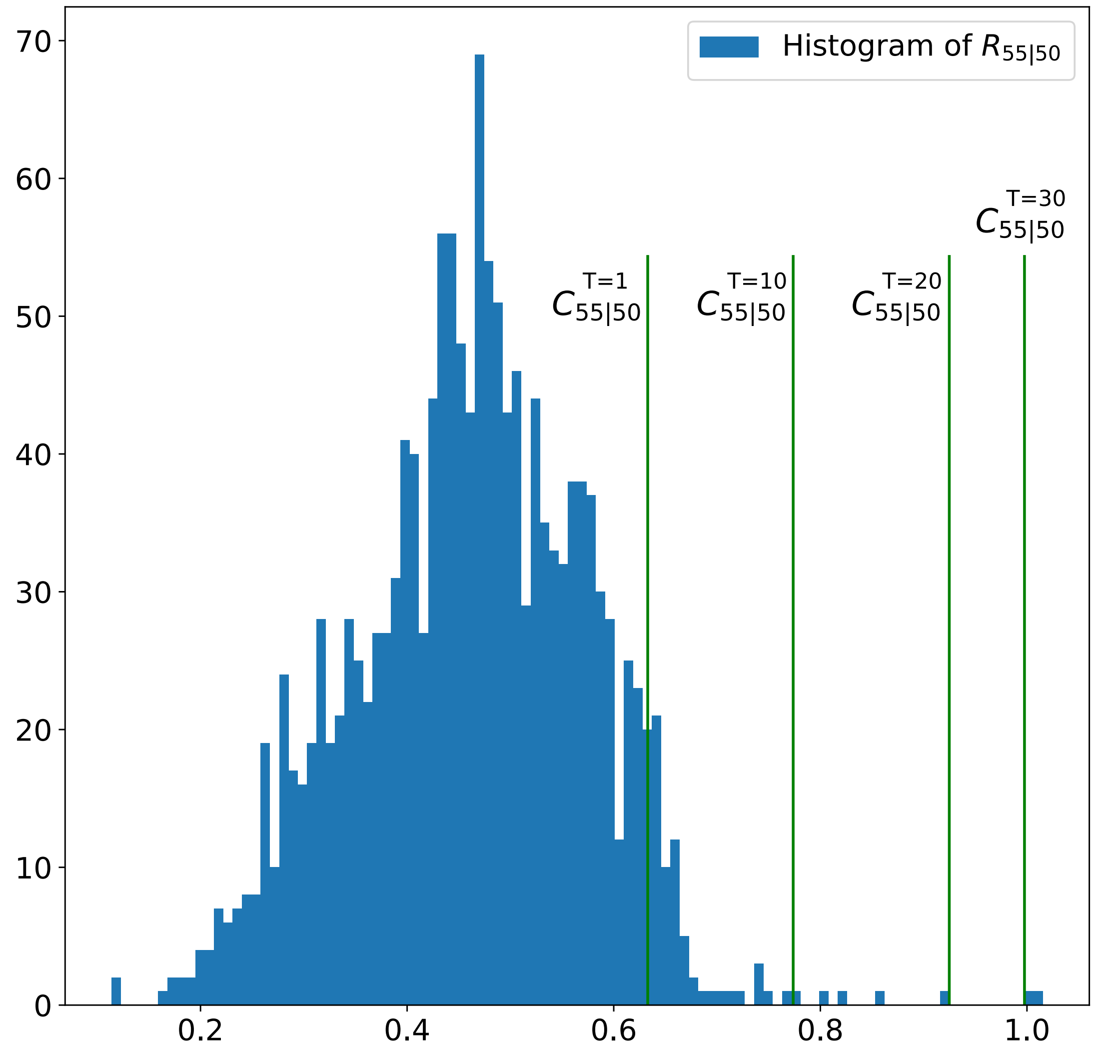

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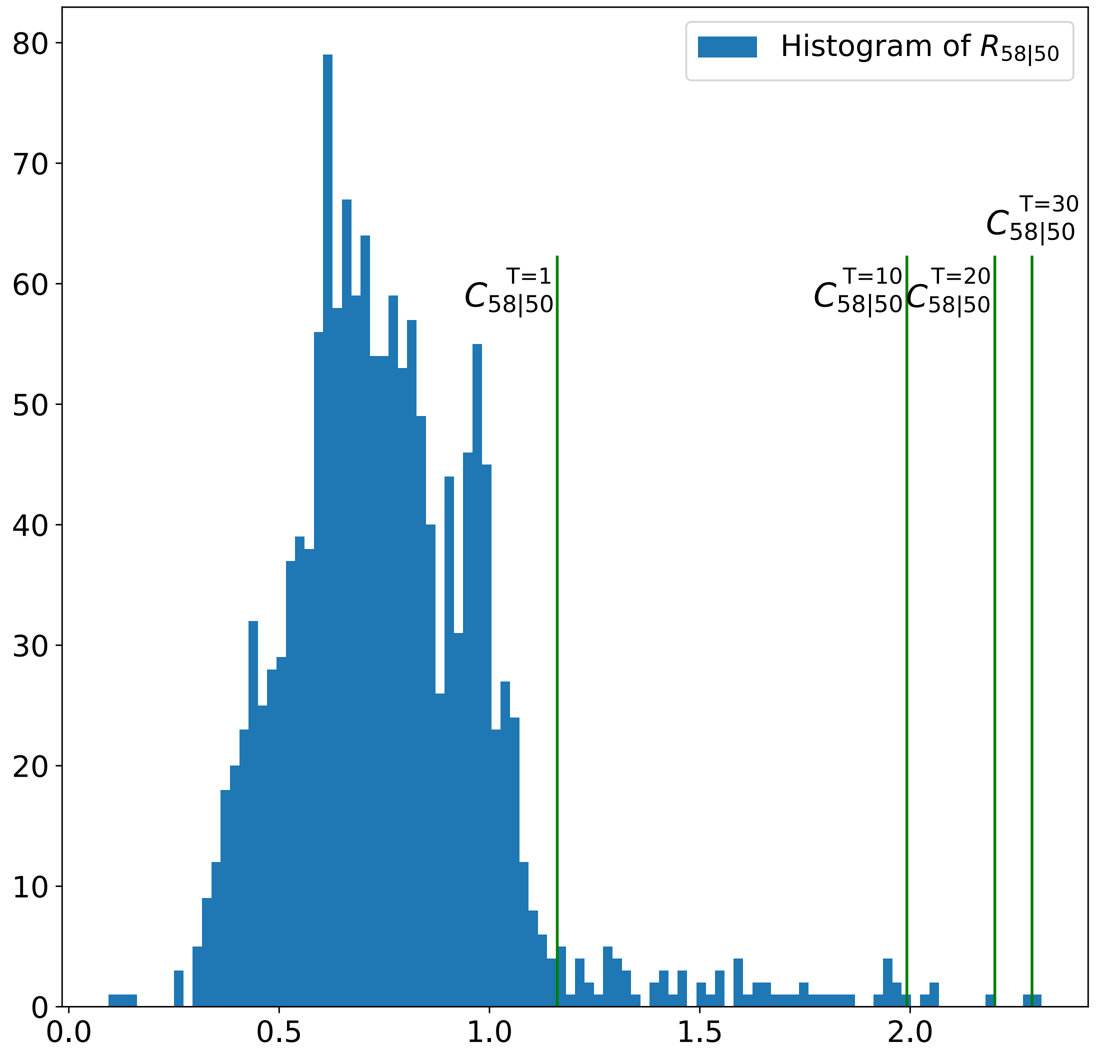

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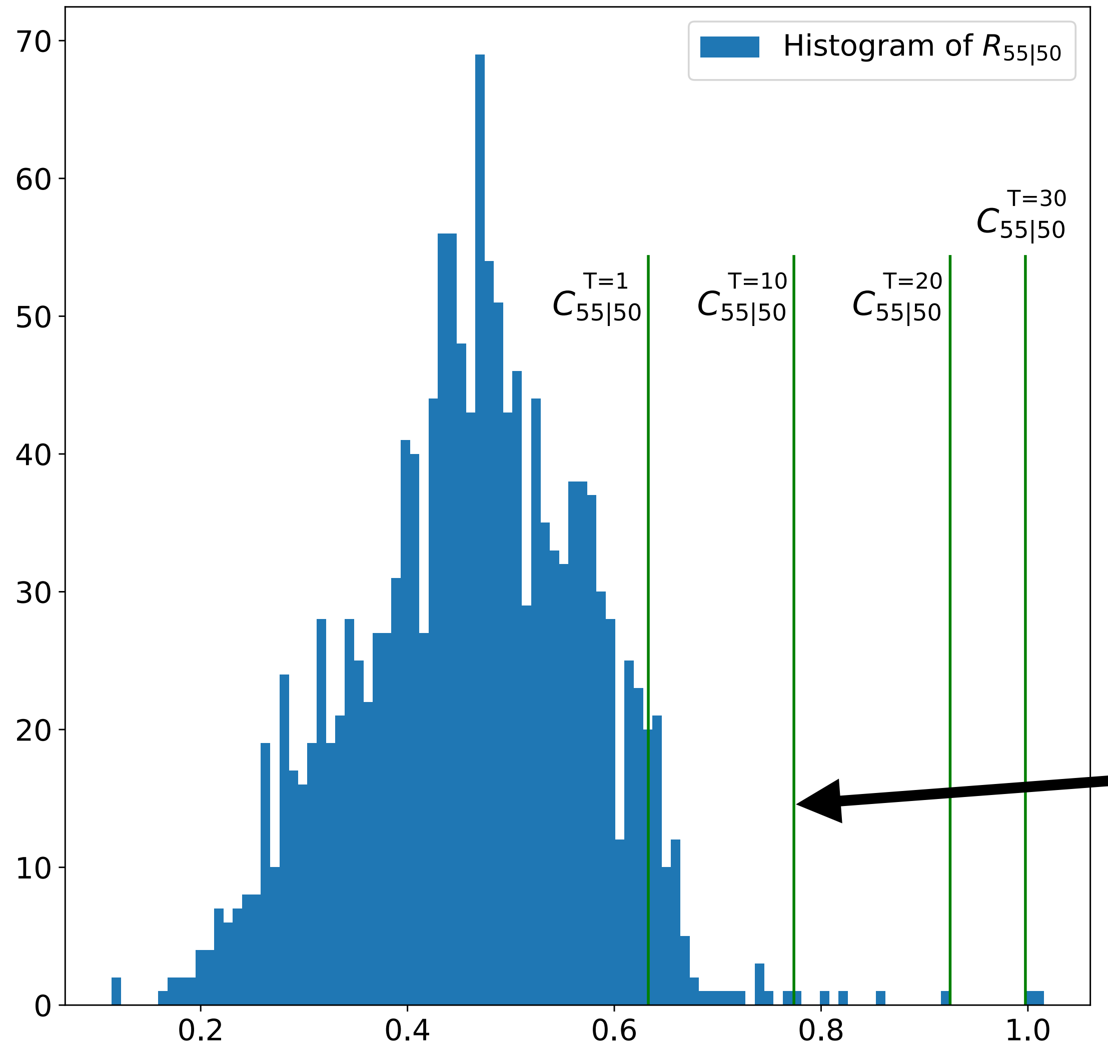

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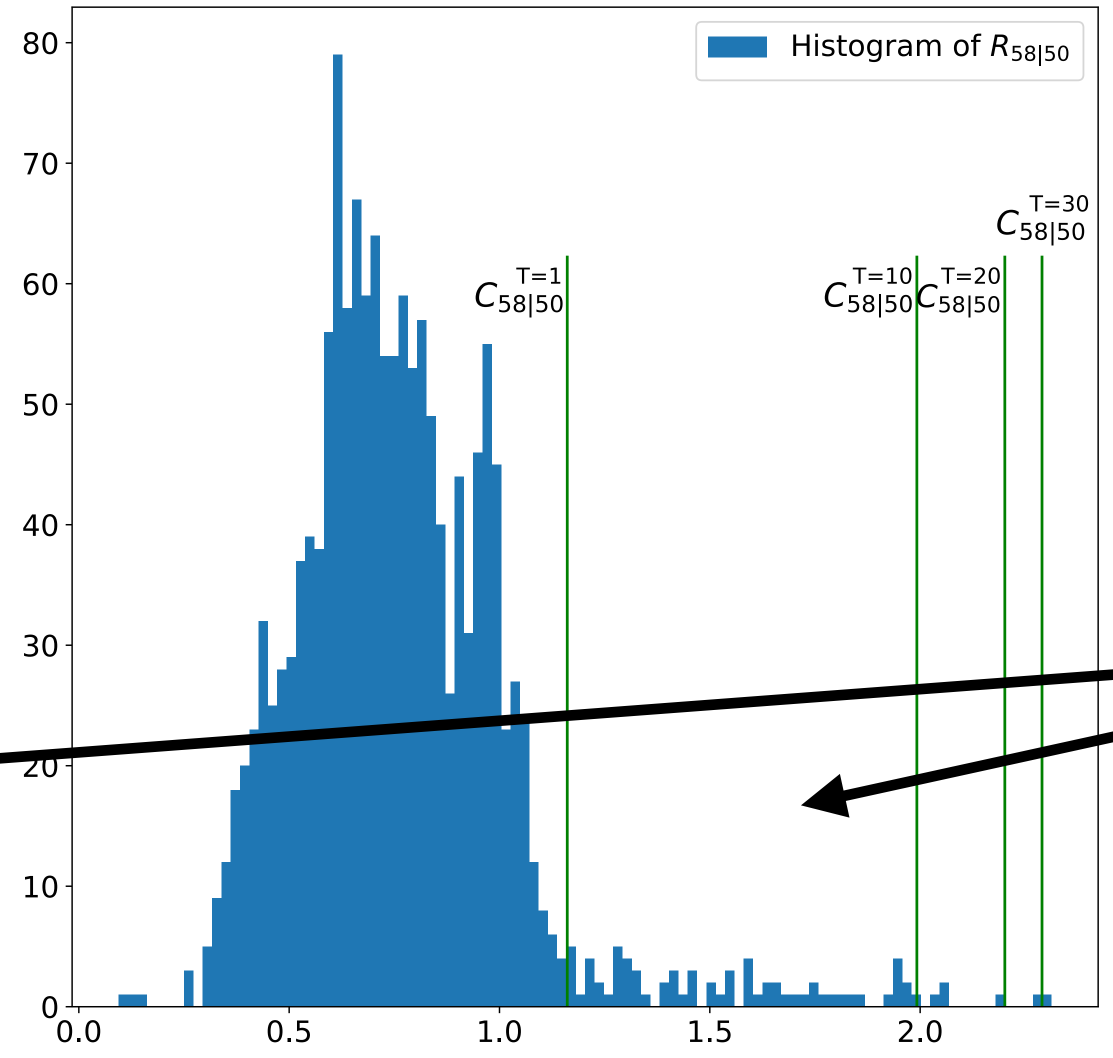

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Size of prediction regions scales with T.

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Efficient uncertainty representations

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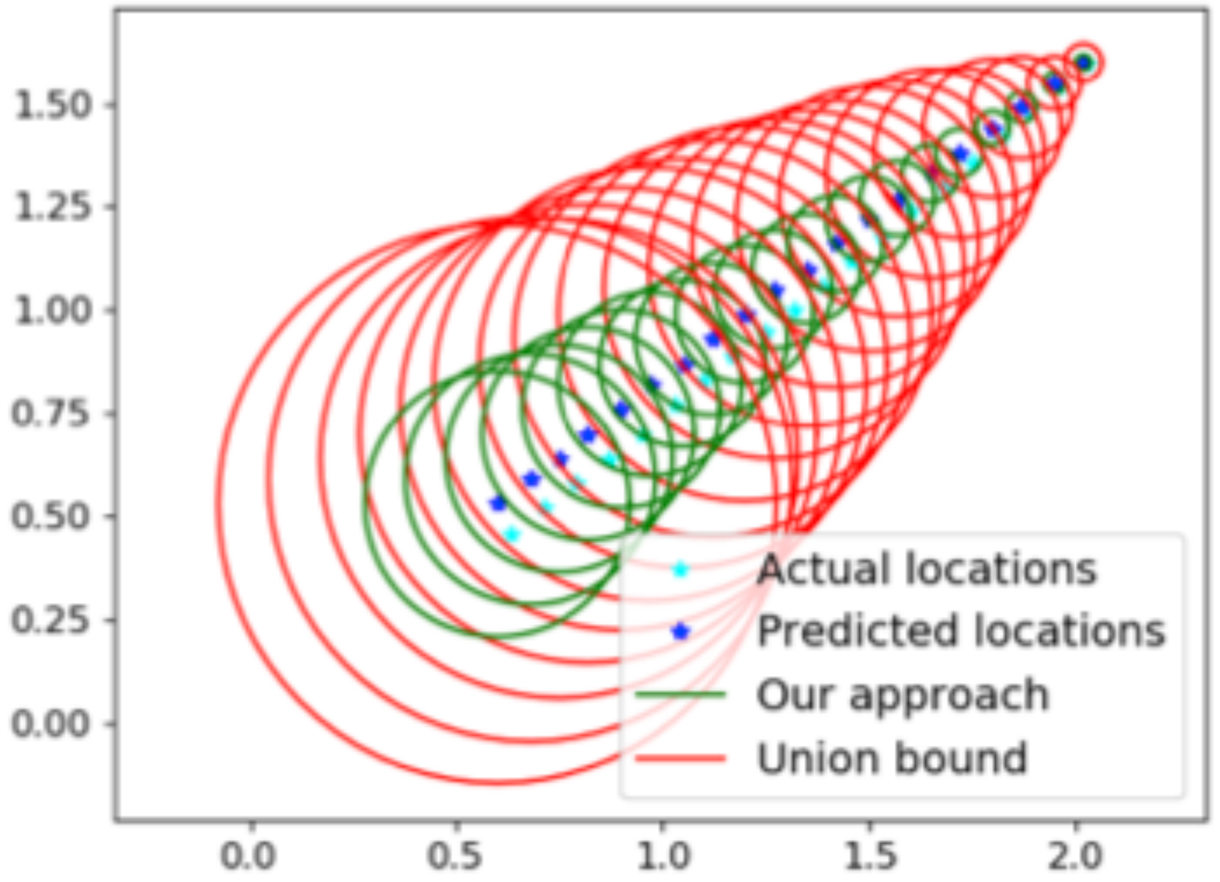
Example:



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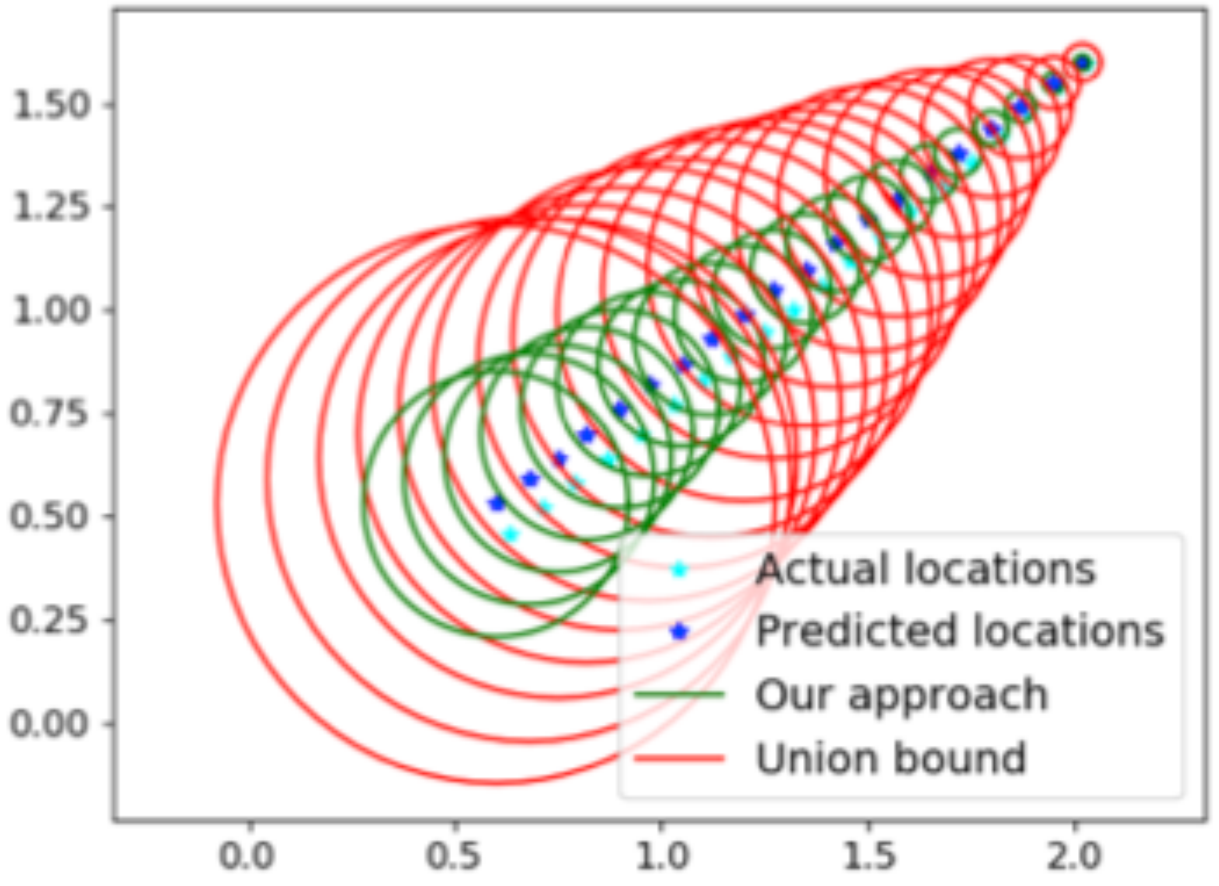
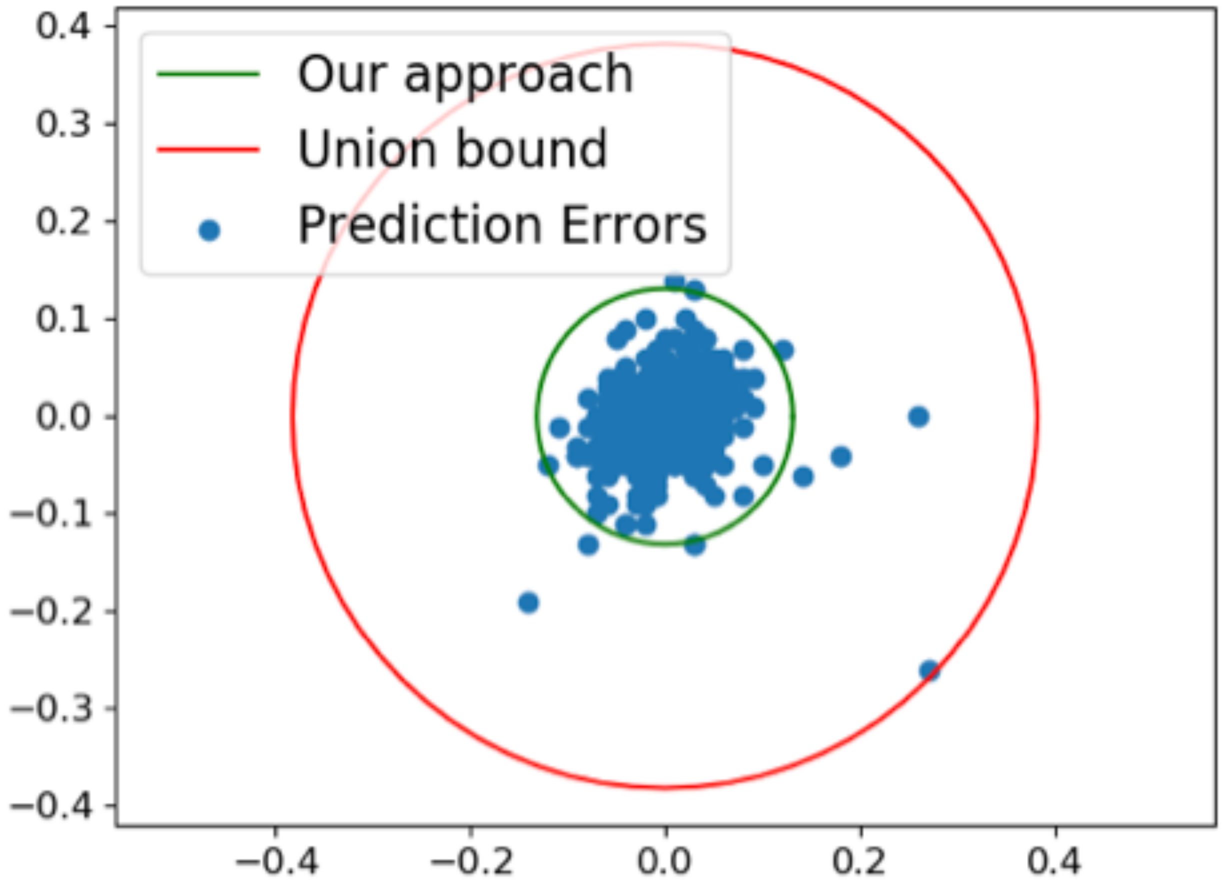
$$R^{(i)} := \max(\alpha_{t+1} R_{t+1}^{(i)}, \ldots, \alpha_T R_T^{(i)}) \longrightarrow \boxed{\text{Prob}(\|y_\tau - \hat{y}_\tau\| \leq C/\alpha_\tau, \ \forall \tau \in \{t+1, \ldots, T\}) \geq 1 - \delta}$$

normalization constant (next slide)

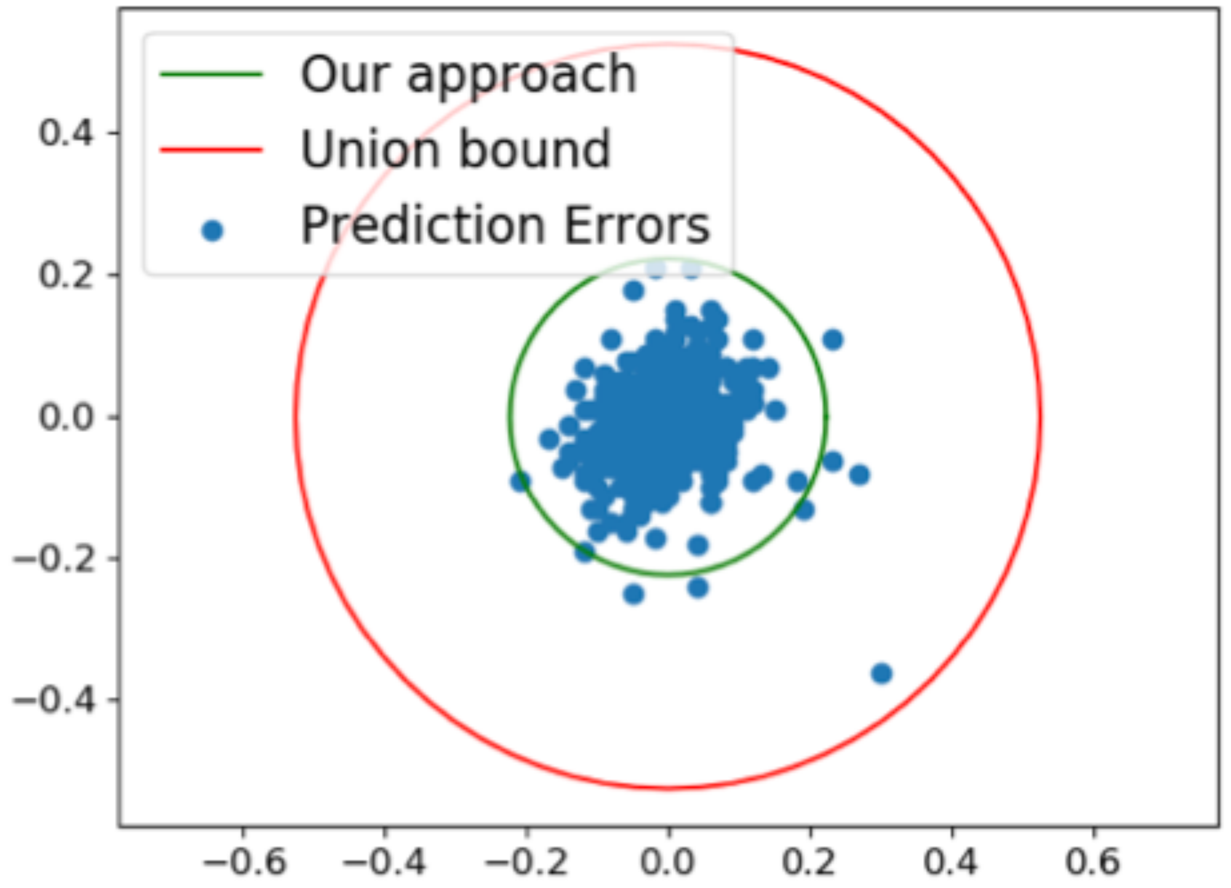
$$C = \text{Quantile}_{1-\delta}(R^{(1)}, \ldots, R^{(k)}, \infty)$$

Example:





1.25 second ahead prediction error



1.8 second ahead prediction error

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[1] Cleaveland, Lee, Pappas, and **Lindemann**, "Conformal Prediction for Time Series using Linear Complementarity Programming", *AAAI*, 2024.

Efficient uncertainty representations

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Uncertainty-aware open-loop control

Open-loop control design:

$$\begin{aligned} & \min_{(u_0,\ldots,u_{T-1})} J(x,u) \\ \text{s.t.}\ \ & x_{\tau+1} = f(x_\tau, u_\tau), && \tau \in \{0,\ldots,T-1\} \\ & \sup_{y_\tau \in \mathcal{B}_\tau} c(x_\tau, y_\tau) \geq 0, && \tau \in \{1,\ldots,T\} \\ & u_\tau \in \mathcal{U}, x_{\tau+1} \in \mathcal{X}, && \tau \in \{0,\ldots,T-1\} \end{aligned}$$

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$c(x_\tau, \hat{y}_\tau) \geq LC_\tau$

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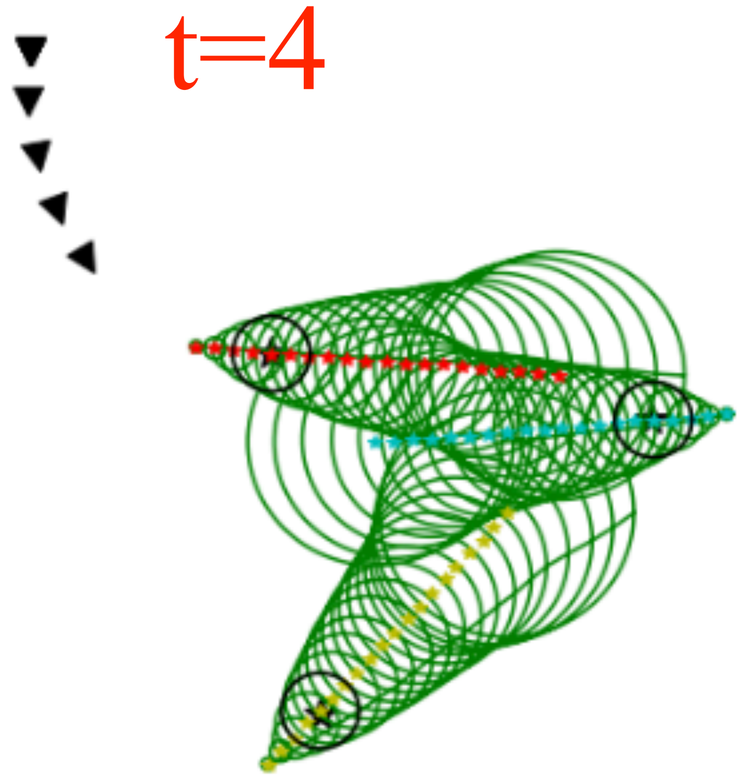
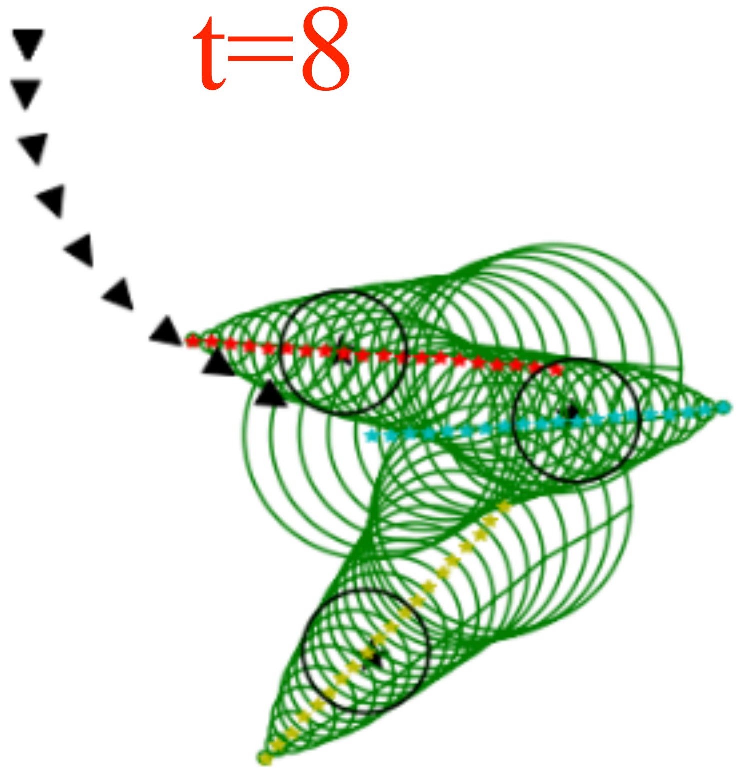
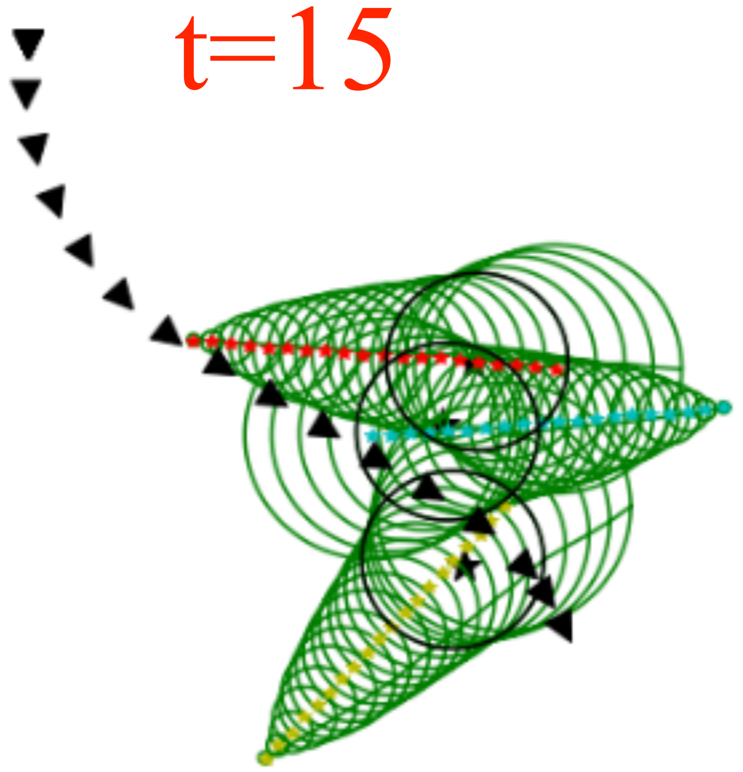
Toy example:



t=4

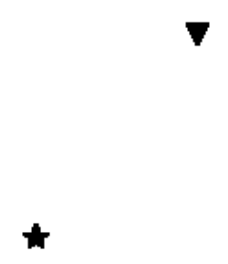


t=8



t=15

Animated:



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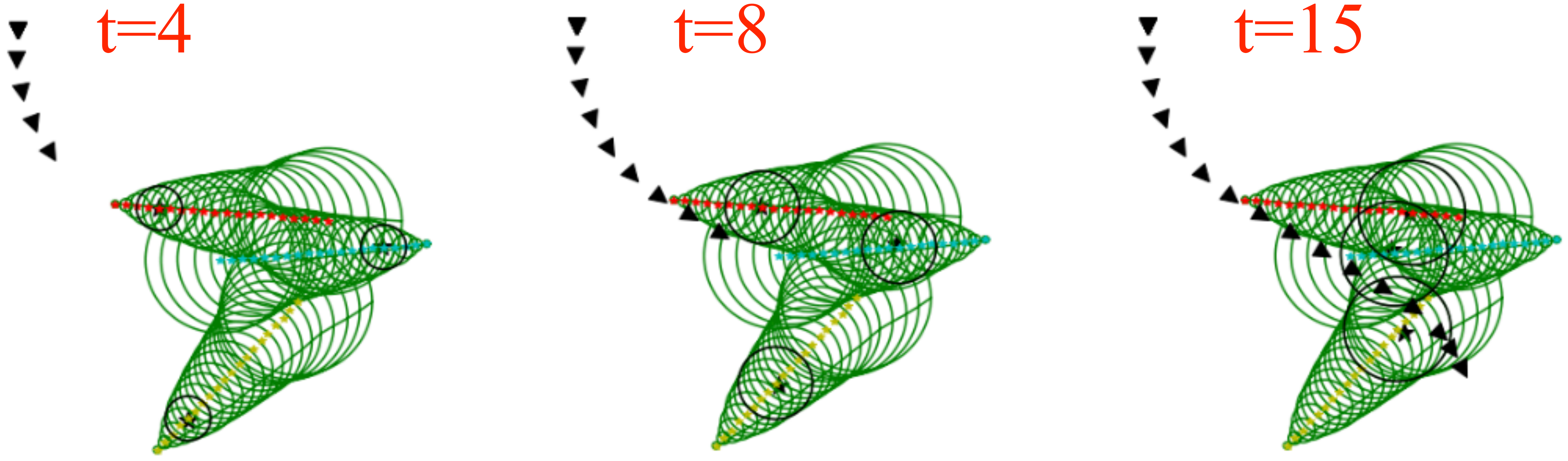

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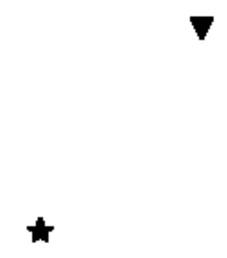

- Potentially conservative and leading to infeasibilities

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[1] Stamouli, **Lindemann**, and Pappas, “Recursively Feasible MPC in Dynamic Environments with Conformal Prediction Guarantees”, *L4DC,* 2024.

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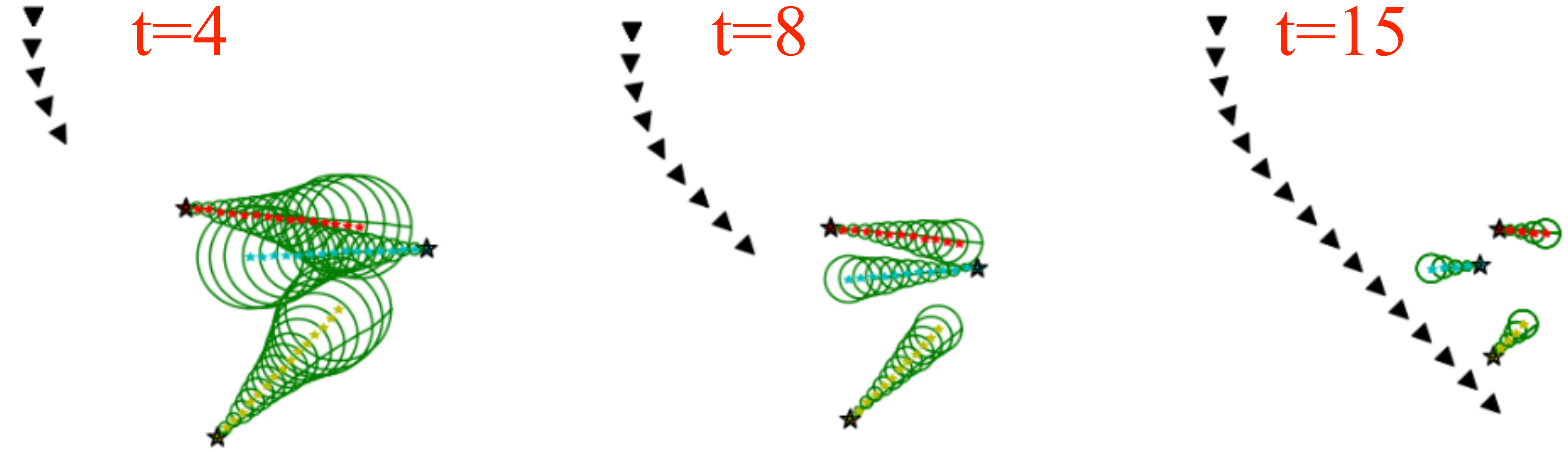

Animated:

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Case Study: CARLA

Results:

- 99.985 % correct one-step ahead predictions (at least 99.83 % expected)
- 99/100 test runs without constraint violation (at most 5 expected)

Comparison with Gaussian Process:

- Largely undercovers

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Temporal Logic Specifications

Control under temporal logic constraints

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- Boolean logic equipped with temporal operators (eventually, always, until)

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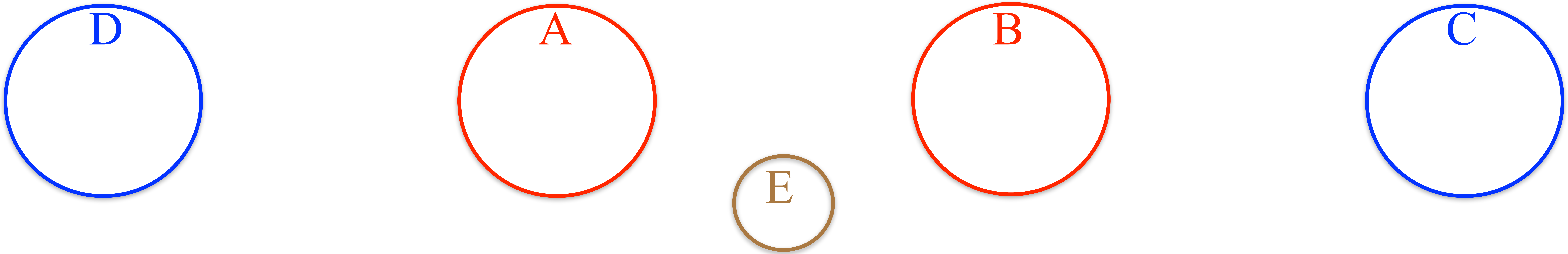

Control under temporal logic constraints

- Boolean logic equipped with temporal operators (eventually, always, until)

1. *Always* avoid E
2. *Every* hour inspect A *and* B
3. *If* battery level is low, *then* recharge *eventually* in C *or* D
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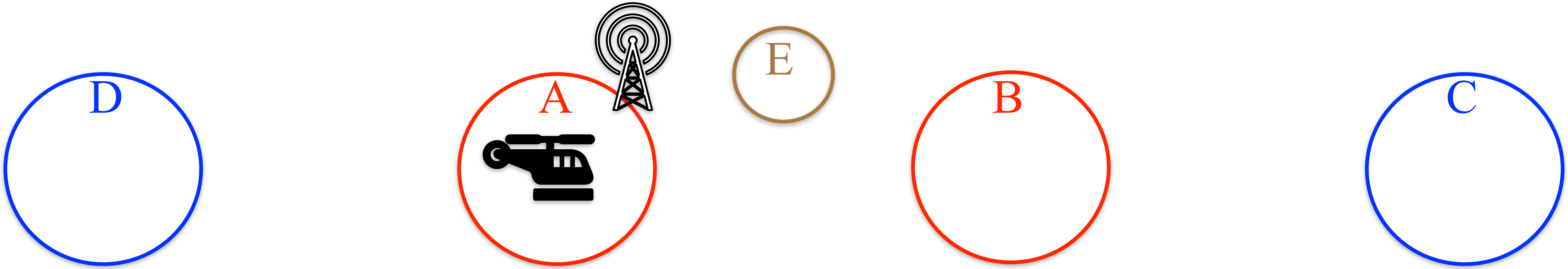

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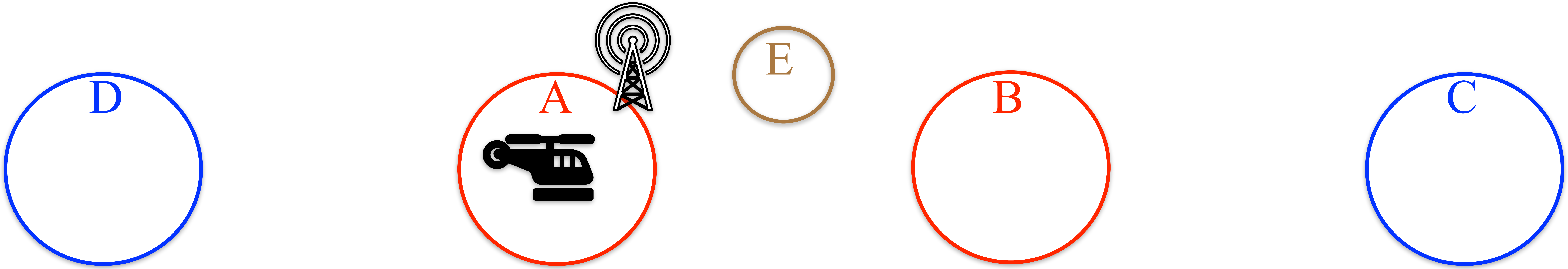


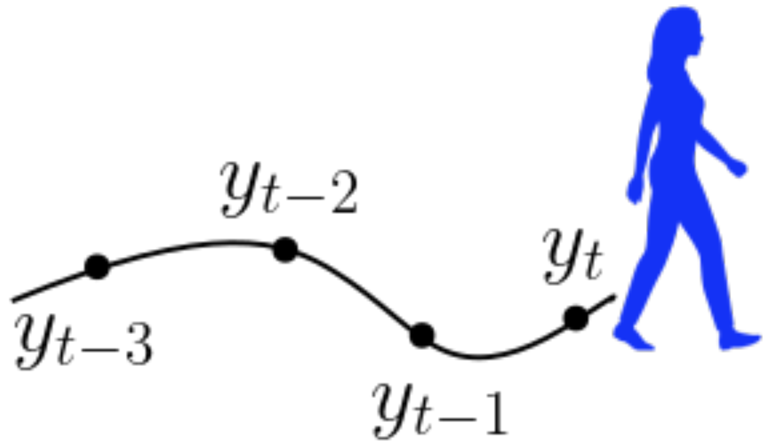
An temporal logic formula ϕ has performance measure ρ^{ϕ}.

Control under temporal logic constraints

Temporal logic task ϕ with **performance measure** $\rho^{\phi}(x, y)$

Dynamics

$x_{t+1} = f(x_t, u_t)$



Yu, Zhao, Yin, and **Lindemann**, “Signal Temporal Logic Control Synthesis among Uncontrollable Dynamic Agents with Conformal Prediction”, TAC *(prep.)*, 2024.

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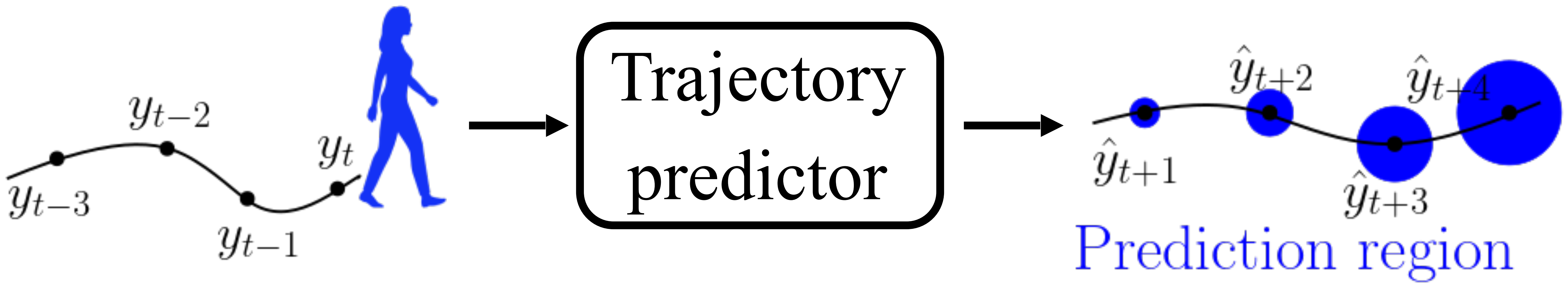

Trajectory predictor

Prediction region

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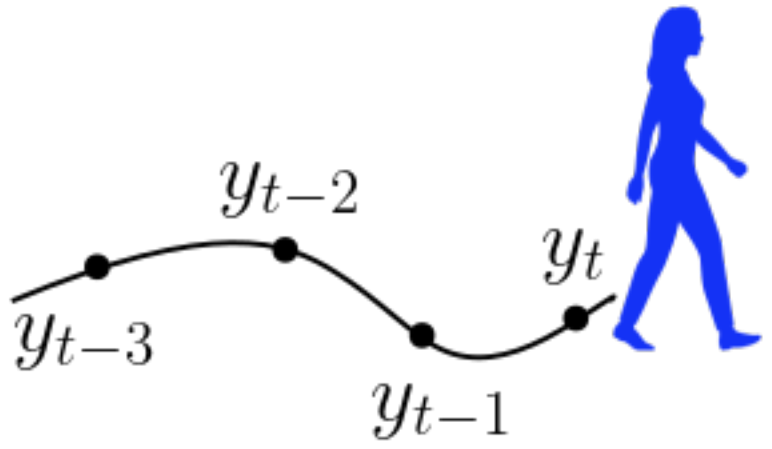

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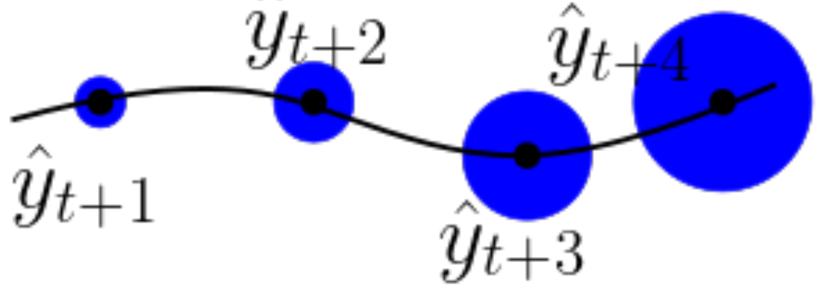

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Robust performance measure $\bar{\rho}^{\phi}(x, \hat{y})$

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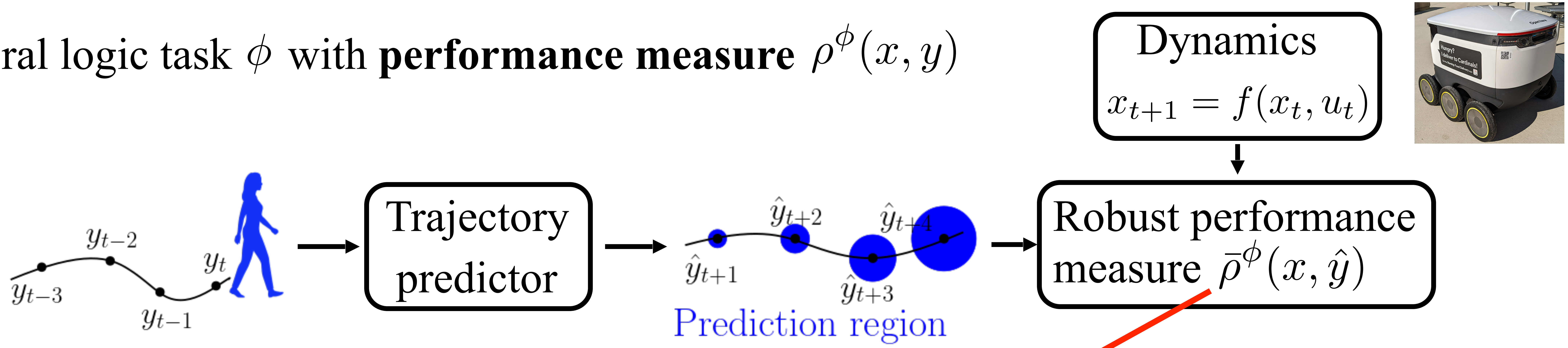

$$\inf_{y_{t+1} \in \mathcal{B}_{t+1}, \ldots, y_T \in \mathcal{B}_T} \rho^{\phi}(x, y)$$

Yu, Zhao, Yin, and **Lindemann**, “Signal Temporal Logic Control Synthesis among Uncontrollable Dynamic Agents with Conformal Prediction”, TAC *(prep.)*, 2024.

Control under temporal logic constraints

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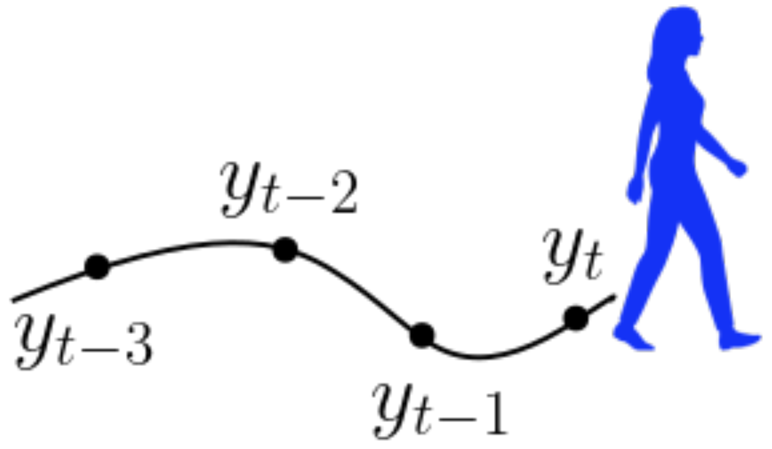
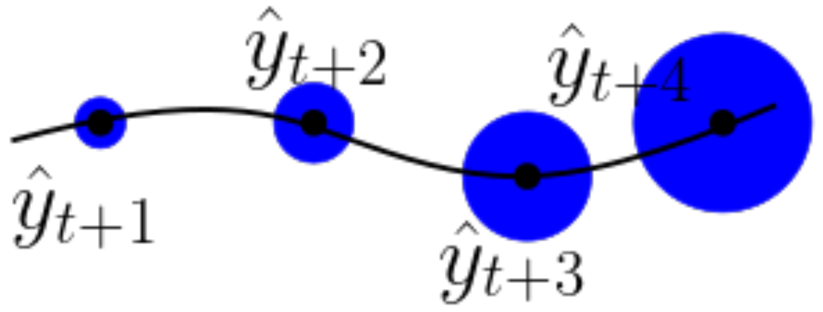
Dynamics

$x_{t+1} = f(x_t, u_t)$



Trajectory predictor



Prediction region

Robust performance measure $\bar{\rho}^\phi(x,\hat{y})$

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$$\mathcal{B}_\tau := \{y \text{ s.t. } \|y-\hat{y}_\tau\| \le C_\tau\}$$

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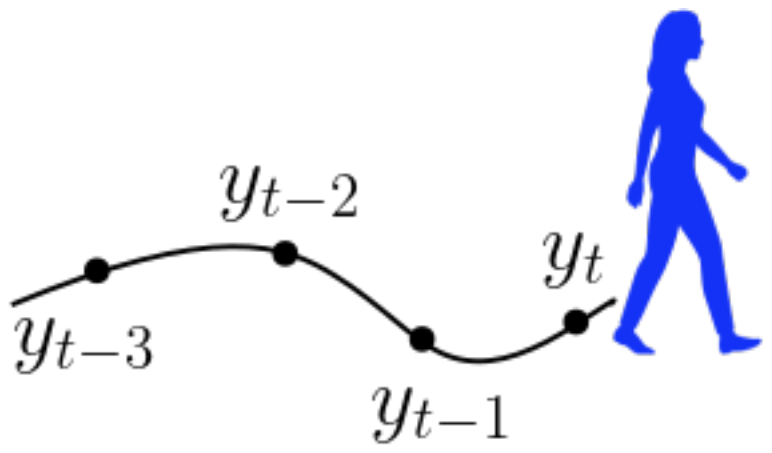

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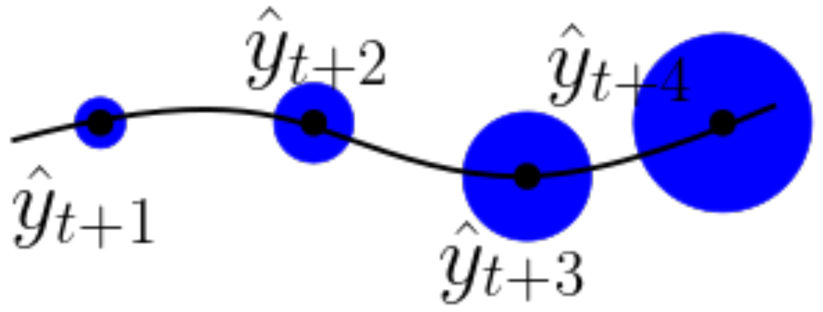

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Control synthesis: $\sup_{u_t,\ldots,u_{T-1}} \inf_{y_{t+1}\in\mathcal{B}_{t+1},\ldots,y_T\in\mathcal{B}_T} \rho^\phi(x, y)$

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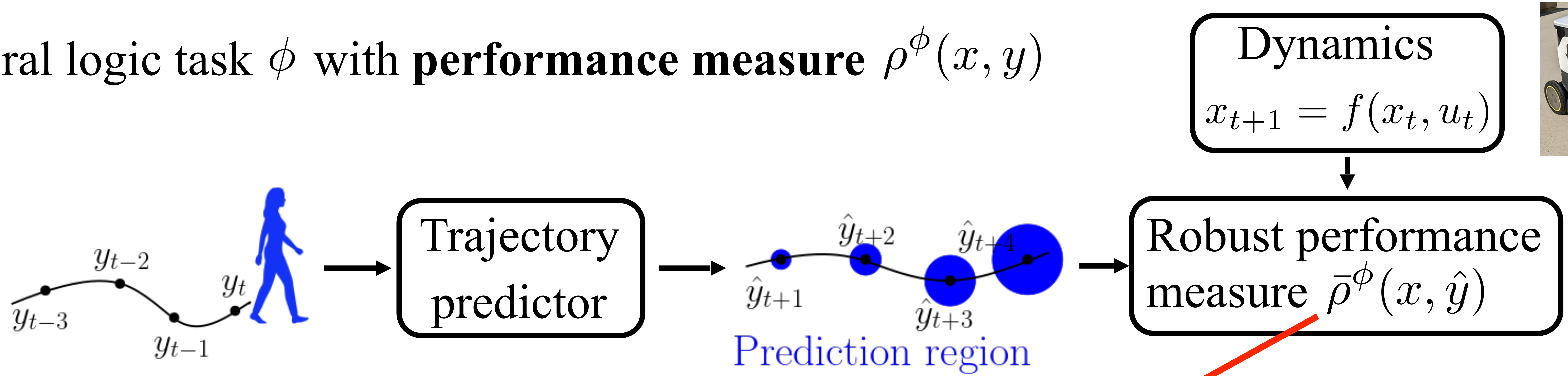

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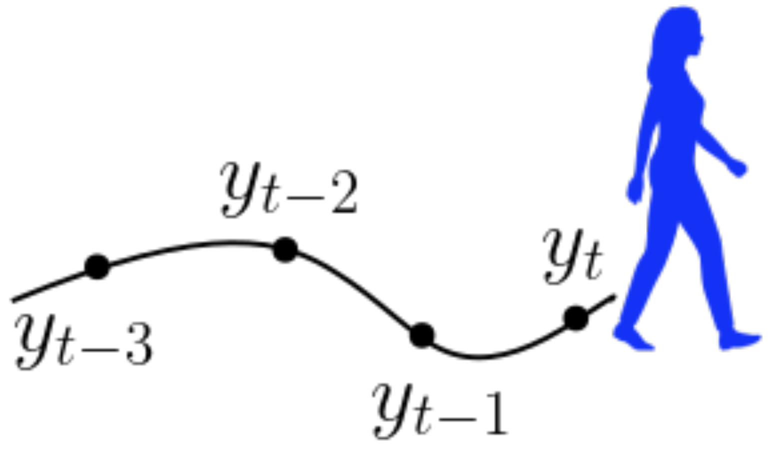

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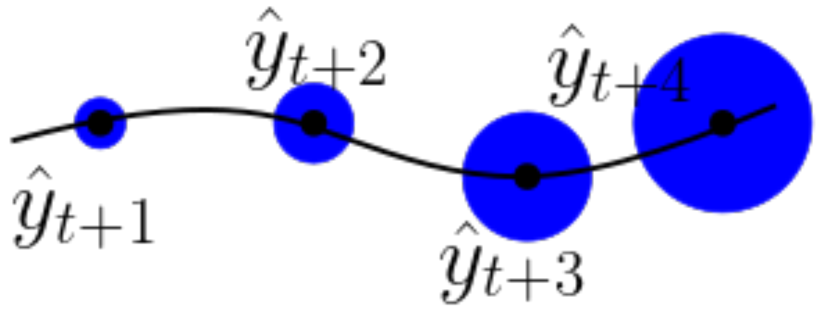

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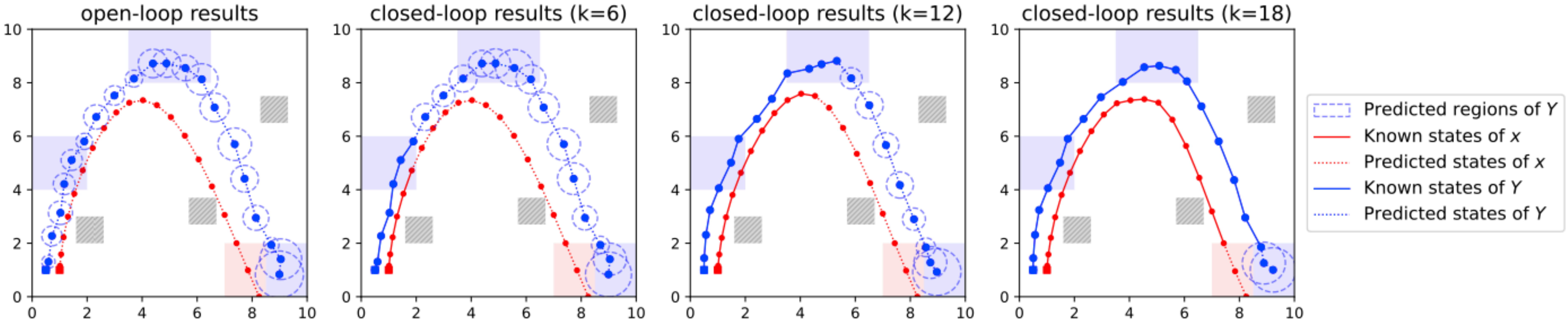
Leader-follower example:



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Distribution Shifts

Safe control when the distribution shifts

- **Design and deployment conditions** may be different, i.e., the distribution may “shift”

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Examples:

changing environments

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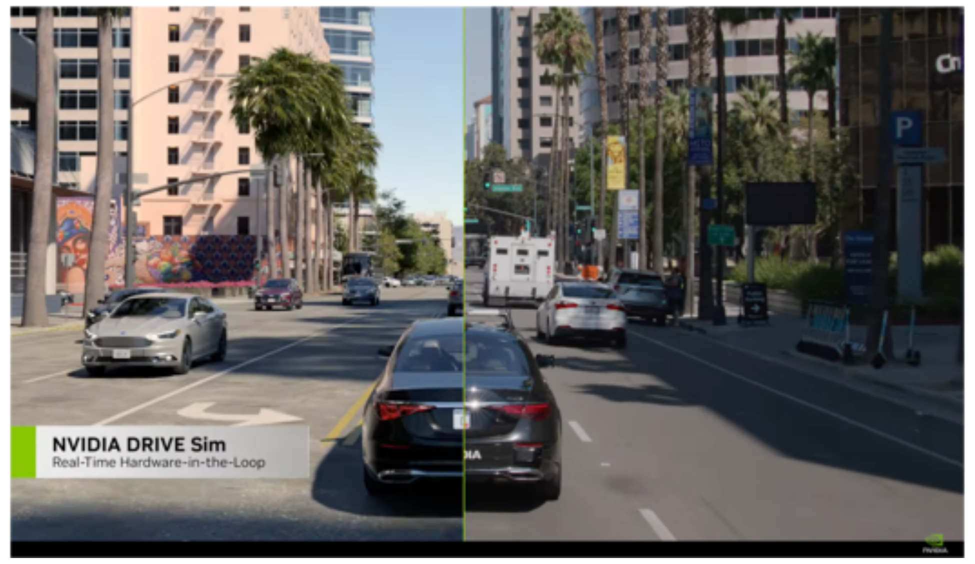

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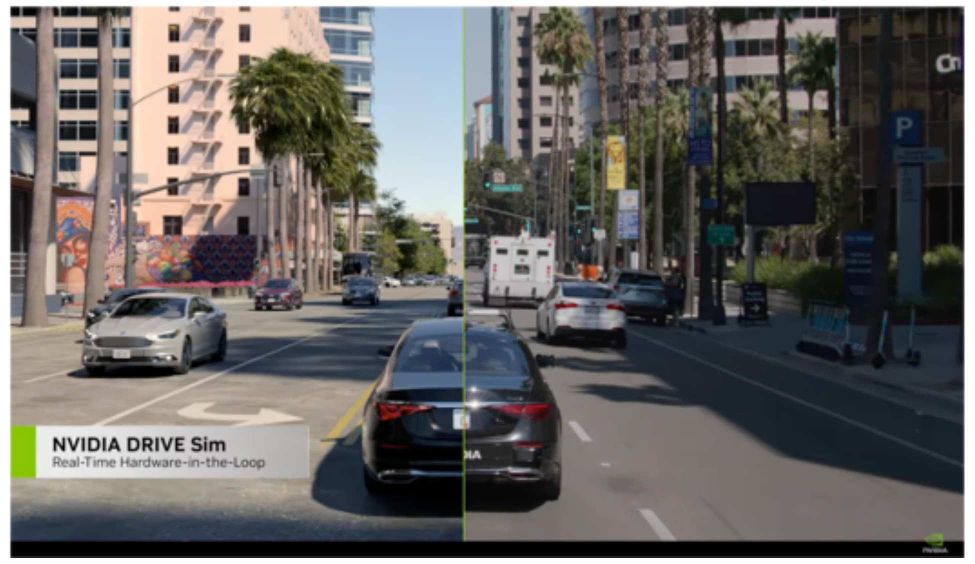

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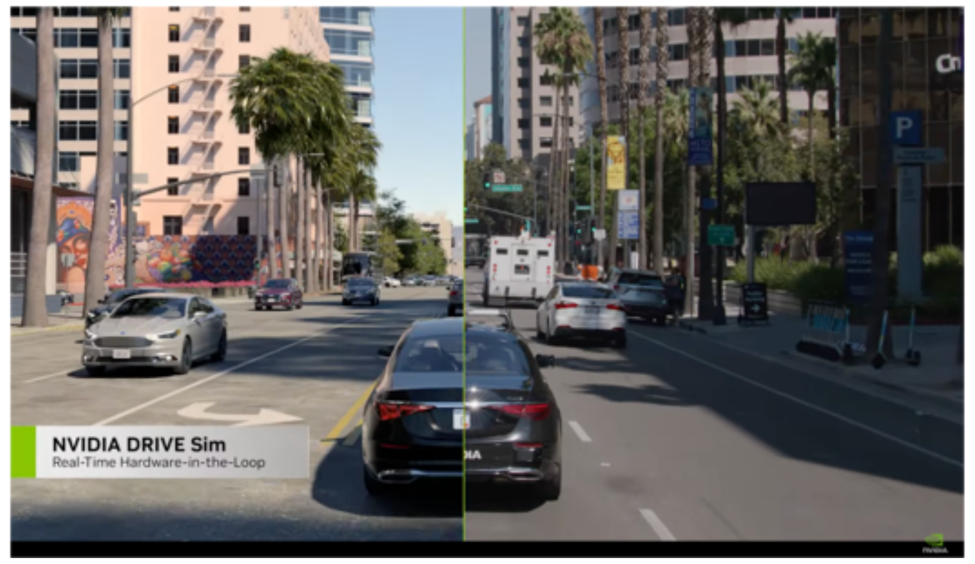

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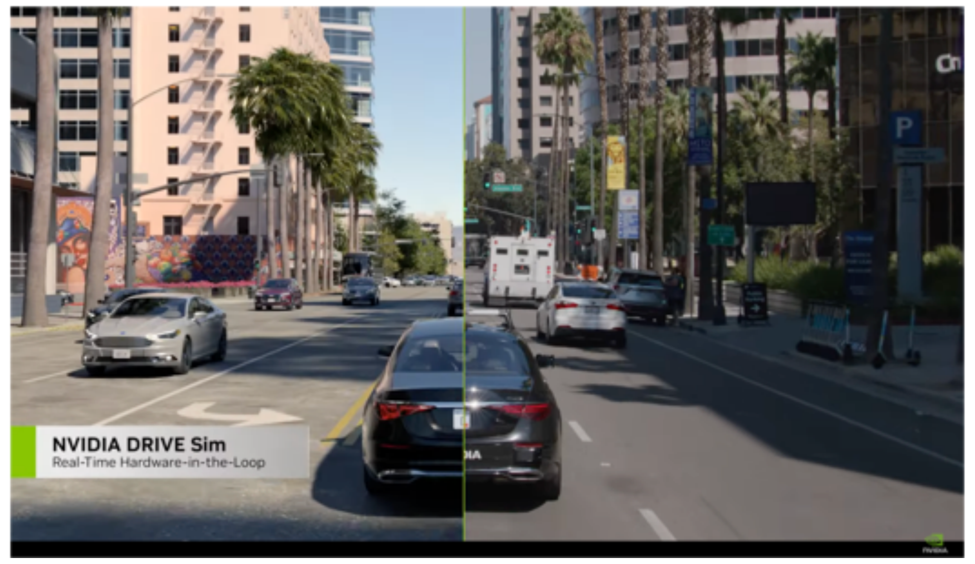

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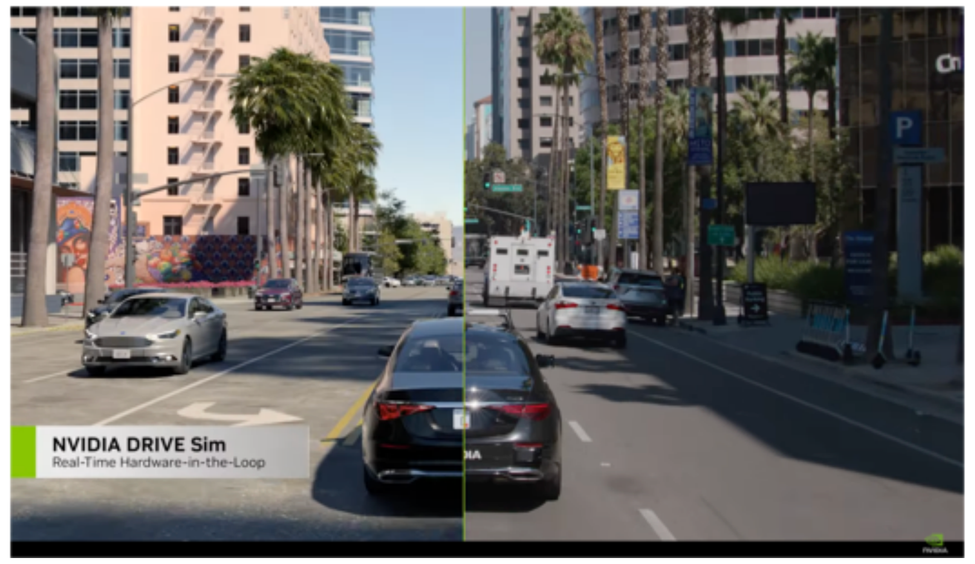

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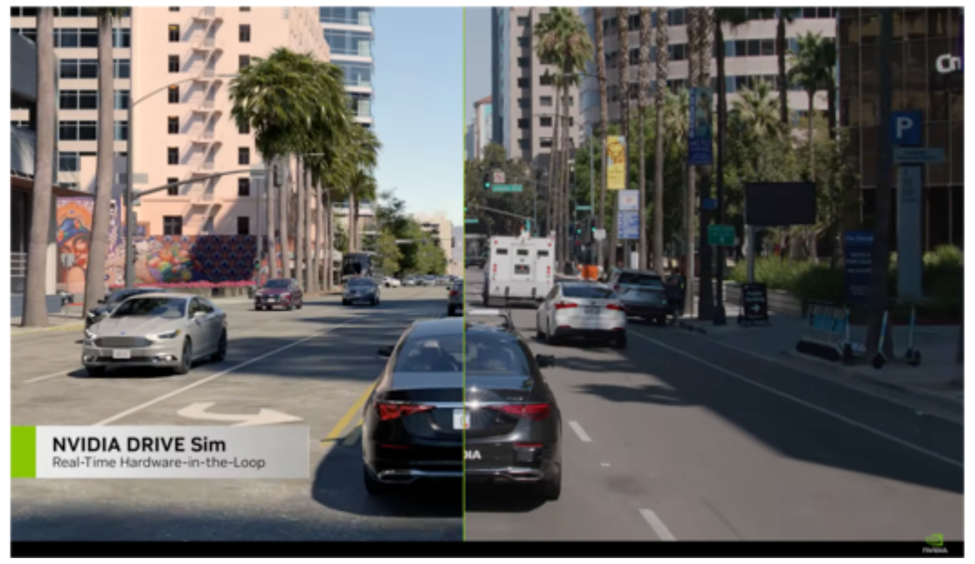

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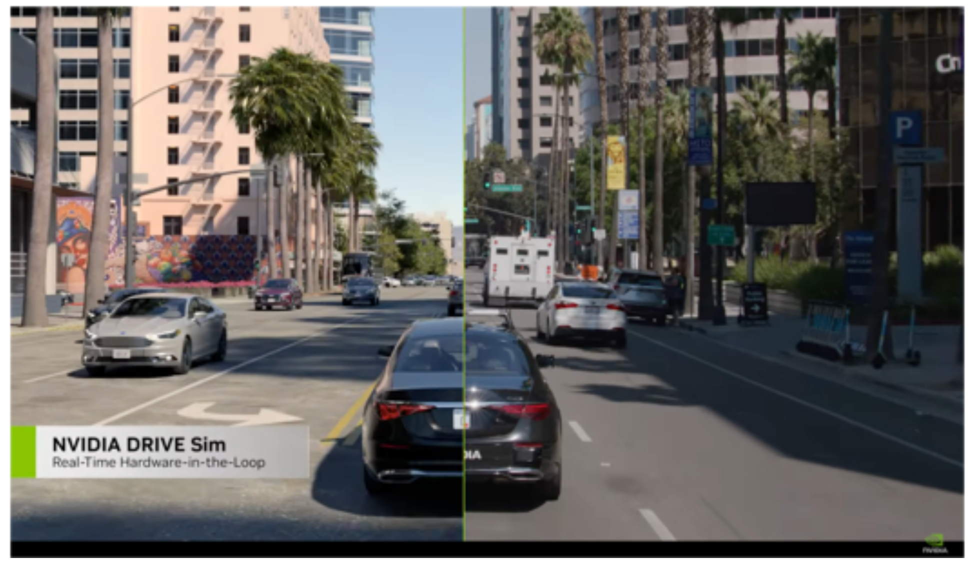

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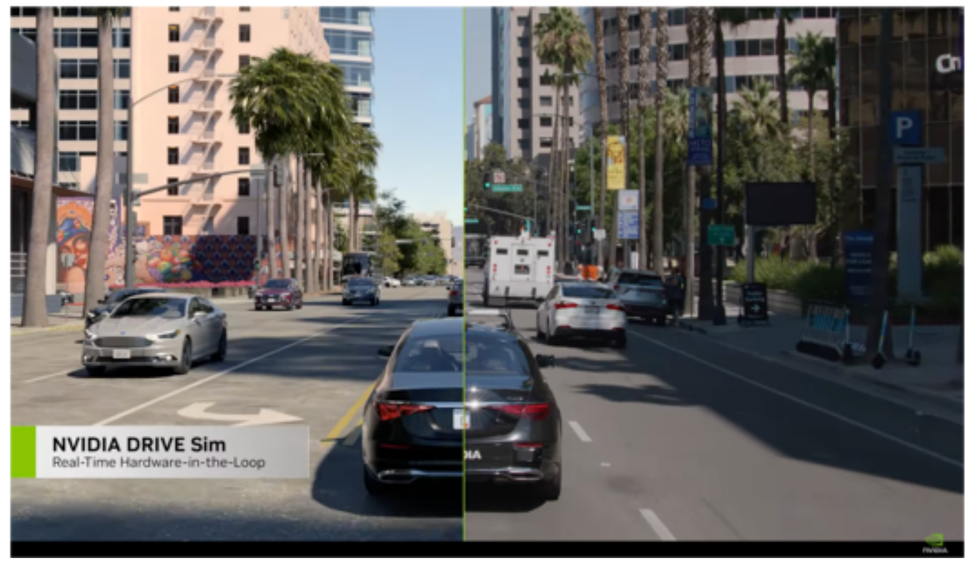

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- Perform control design with robust conformal prediction regions[2]

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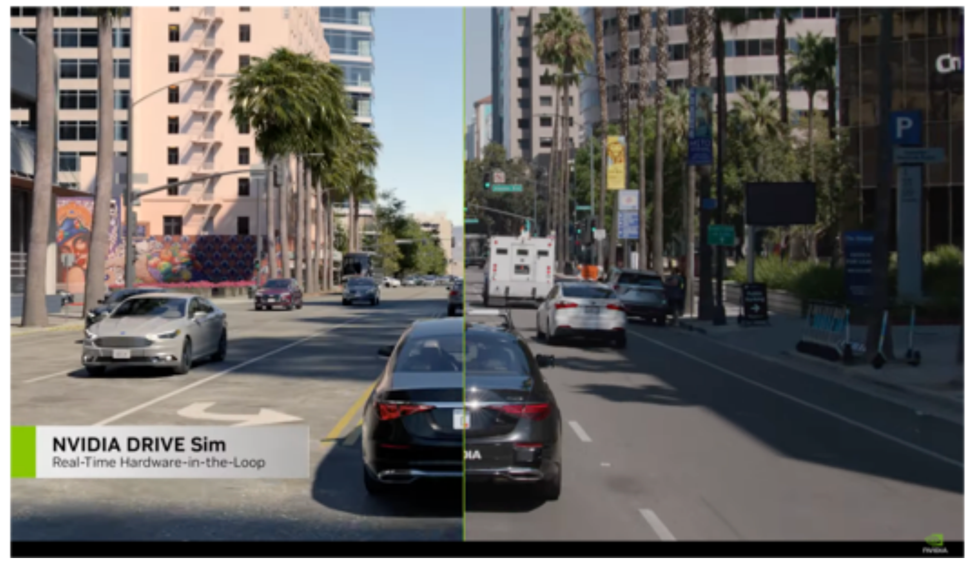

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Best Paper Award Finalist

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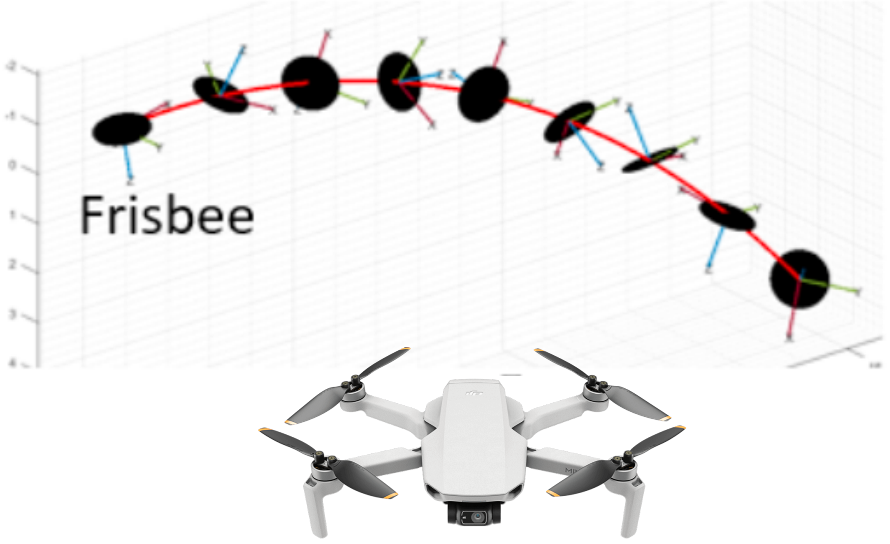

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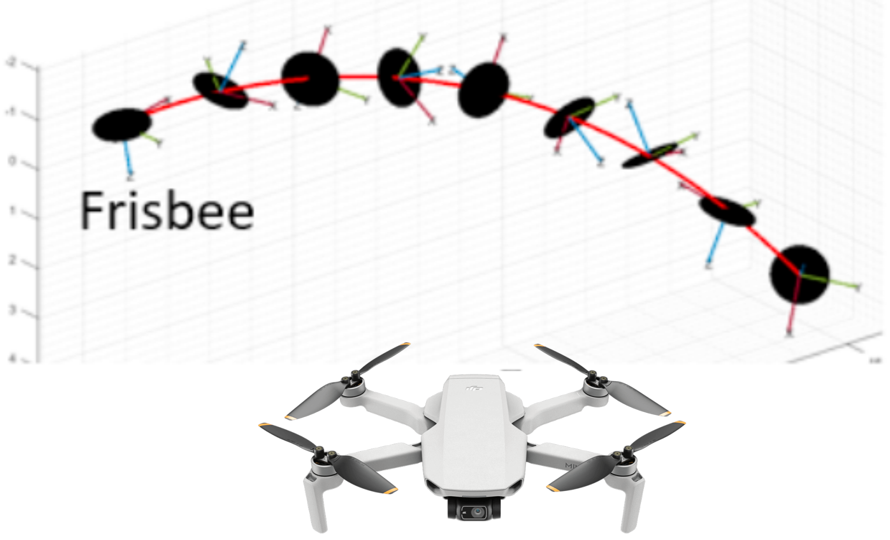

Case	δ	0.025			0.05		
	UQ method	Proposed	Wei et al. (2022)	w/EKF	Proposed	Wei et al. (2022)	w/EKF
Frisbee	%Succ.	99.2	100	100	100	100	100
w/drag	$\bar{d}_{min}$	2.91	14.2	5.27	2.74	4.97	4.25

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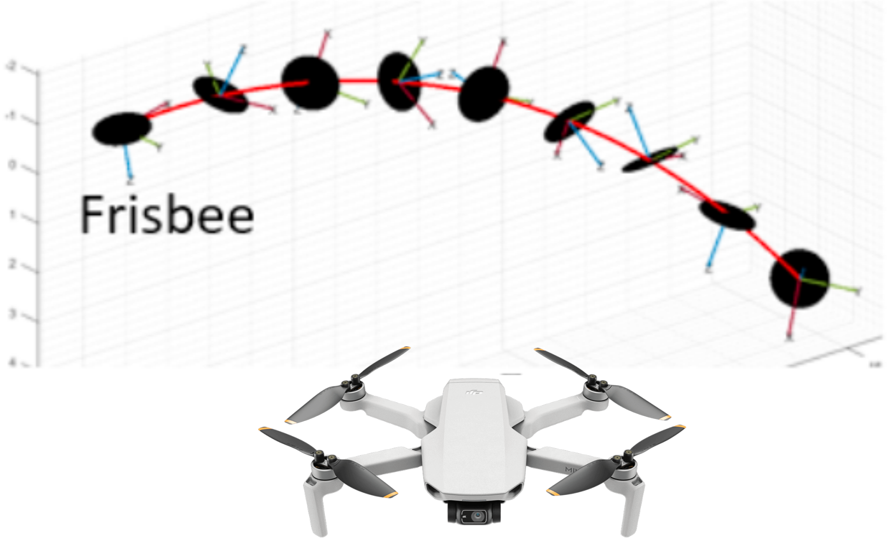

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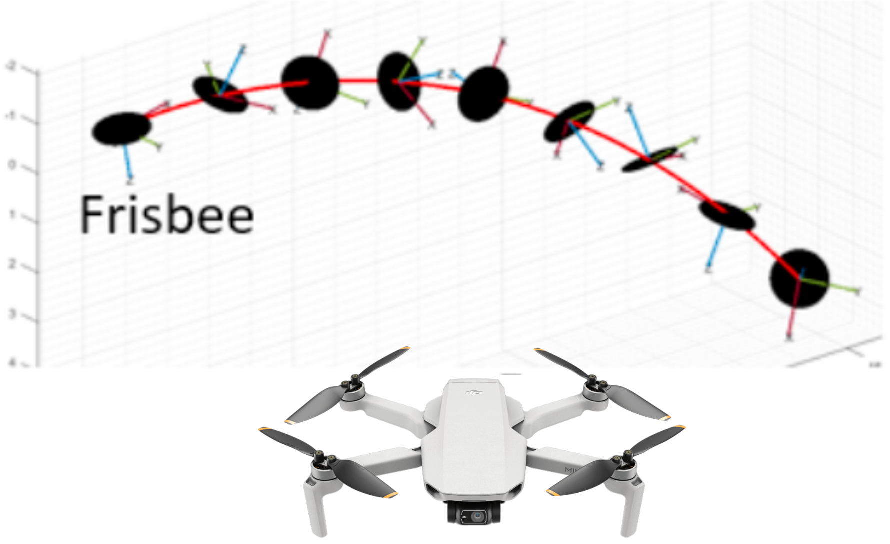

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Selected References

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Main Collaborators


Matthew Cleaveland


Yiqi Zhao


Xinyi Yu


George Pappas


Jyotirmoy Deshmukh


Anushri Dixit