Neural Operators That Stabilize **Infinite-Dimensional** Systems

Miroslav Krstic

★ 2026 IFAC WC workshop: ***Trustworthy Data-Driven Control*** ★

Neural Operators That Stabilize

Infinite-Dimensional Syste

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How to control PDEs
that are NONLINEAR
+
have totally UNKNOWN
parameters

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with

100% "TRUST" (proofs)?

Feedback linearization: from finite to infinite dimension

ODEs (Isidori-Krener-GG-M, 1981; Hunt-Su-Meyer, 1983): $\boxed{\textit{done}\text{, 45 years ago}}$

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Hyperbolic PDEs: $u_t = u_x + F[u]$: *open until now*

Hyperbolic plant class — a benchmark in ∞ dimension

$$\boxed{u_t = u_x + F[u]}, \qquad u(1,t) = U(t)$$

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with F a spatial Volterra series

$$F[u](x,t) = \sum_{n\geq 2} \int_0^x \int_0^{\xi_1} \cdots \int_0^{\xi_{n-1}} f_n(x,\xi_1,\ldots,\xi_n)\, u(\xi_1,t)\cdots u(\xi_n,t)\, d\xi_n \cdots d\xi_1$$

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"Taylor series for causal operators" (Volterra 1887; Brockett 1976; Krener 1978)

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1. **exact** feedback linearization — but: controller = infinite series
2. **truncated**/approximate FL — but: kernel re-solve & real-time spatial integration
3. controller **learned** offline as neural operator — but: plant assumed known
4. **adaptive** + neural operator — *learns the plant online (my dream since 1997)*

Exact FL

Truncated controller

Neural operator for full feedback law

Adaptive + NO controller

1

The **Nonlinear Control** Half

Exact FL

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Backstepping target

Construct nonlinear transform & *target system* (pure transport/delay)

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and feedback

$$\boxed{U(t) = K[u](1,t)} \qquad w(1,t) = 0 \qquad \Longrightarrow \quad \text{vanishes in } t = 1$$

K = Volterra series, just like F:

$$K[u](x,t) \;=\; \sum_{n\geq 2} \int_0^x \int_0^{\xi_1} \cdots \int_0^{\xi_{n-1}} k_n(x, \xi_1, \ldots, \xi_n)\, u(\xi_1, t) \cdots u(\xi_n, t)\, d\xi_n \cdots d\xi_1$$

Matching identity

Kernels must satisfy ∞ many Goursat hyperbolic PDEs

$$\boxed{\left(\partial_x + \sum_{i=1}^{n} \partial_{\xi_i}\right) k_n = -f_n + \sum_{m=2}^{n} B_n^m[k_{n-m+1}, f_m]} \qquad k_n|_{\xi_n=0} = 0$$

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Integral form:

$$k_n(x, \xi_1, \ldots, \xi_n) = -\int_0^{\xi_n} \Big[f_n - \sum_{m=2}^{n} B_n^m[k_{n-m+1}, f_m] \Big](x-\xi_n+s,\ \xi_1-\xi_n+s,\ \ldots,\ \xi_{n-1}-\xi_n+s,\ s)\, ds$$

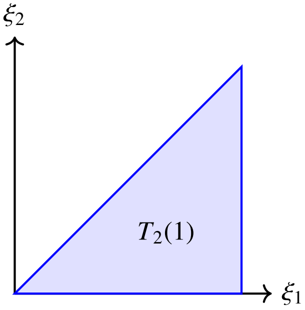
What does this make the controller $U(t) = K[u](1,t)$?

infinite sum of

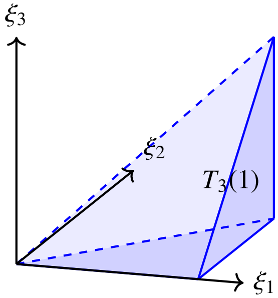
integrals nested **infinitely**

with gains $\{k_n\}$ on simplices of dimension that grows **infinite**

Kernels on simplices

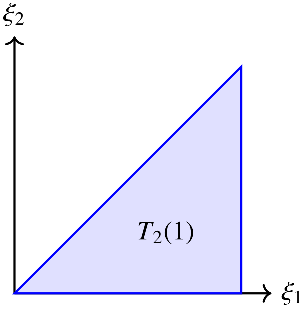


$k_2(\xi_1, \xi_2)$



$k_3(\xi_1, \xi_2, \xi_3)$

Kernels on simplices



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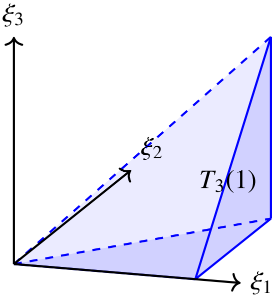

$k_3(\xi_1, \xi_2, \xi_3)$

n-simplex $\quad T_n(1) = \{0 \leq \xi_n \leq \cdots \leq \xi_1 \leq 1\}$ of volume $\dfrac{1}{n!}$

Dense technical core of the talk

(2 slides)

$K[u]$ converges

Plant (analyticity) assumption: $$\|f_n\|_\infty \;\leq\; D_f \frac{n!}{\rho_f^{n-1}}$$

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Lemma (Kernel bound)

$$\boxed{\|k_n\|_{L^2(T_n(1))} \;\leq\; D_f \, e^{\Upsilon_K} \frac{\sqrt{n!}}{\rho_K^{n-1}}} \qquad \Upsilon_K := 1 + \frac{4}{3} D_f \,, \qquad \rho_K := \frac{\rho_f}{2\, e^{\Upsilon_K}}$$

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Lemma (Explicit bound on backstepping operator K)

Class $\mathcal{K}_\infty$ in $\|u\|_{L^2}$ on $[0, \rho_K)$:

$$\boxed{\|K[u]\|_{L^2} \le \kappa_2 \frac{\|u\|_{L^2}^2}{(1 - \|u\|_{L^2}/\rho_K)^2}} \qquad \kappa_2 := \frac{8\, D_f\, e^{2\Upsilon_K}}{\rho_f}$$

Exponential stabilization by exact FL

Theorem (Closed-loop exponential stability in L^2)

$\forall$ *decay rates* $\lambda > 0$, $\exists$ *explicit* $C_1, C_2 > 0$ *s.t., if*

$$\|u_0\|_{L^2} < C_1$$

then

$$\boxed{\|u(\cdot, t)\|_{L^2} \;\leq\; C_2\, e^{-\lambda t}\, \|u_0\|_{L^2}} \qquad t \geq 0$$

Exact FL

Truncated controller

Neural operator for full feedback law

Adaptive + NO controller

Infinite series — show stopper for implementation

Truncate at order N:

$$U(t) = K_N[u](1,t) = \sum_{n=2}^{N} \int_{T_n(1)} k_n \prod_i u(\xi_i, t)\, d\xi$$

What can "break" under truncation?

Under **truncated** feedback: the target gets boundary residual:

$$w_t = w_x\,, \qquad \boxed{w(1,t) \;=\; K[u](1,t) - K_N[u](1,t)} \;\neq 0$$

Residual decays in N

$$\boxed{\left|K[u](1) - K_N[u](1)\right| \;\le\; \varepsilon_N(\|u\|_\infty) \;=\; D_f\, e^{\Upsilon_K}\, \rho_K \cdot \frac{(\|u\|_\infty/\rho_K)^{N+1}}{1 - \|u\|_\infty/\rho_K}}$$

For $\|u\|_\infty < \rho_K$, *geometric decay of residual in* N

Truncated FL asymp. stabilization

Theorem (Stability under K_N)

$\forall$ *truncation orders* $N \geq 2$, *on a ball in* L^∞, $\exists\, \beta \in \mathcal{KL}$, *s.t., under* $U(t) = K_N[u](1,t)$,

$$\boxed{\|u(\cdot,t)\|_\infty \;\leq\; \beta\left(\|u_0\|_\infty, t\right)}$$

Well-posedness in closed loop

Mild solution form along characteristics:

$$\boxed{u(x,t) = \int_{\max\{0,\ t-(1-x)\}}^{t} F[u(\cdot,s)](x+t-s)\,ds + \begin{cases} u_0(x+t)\,, & t \in [0,\ 1-x] \\ K_N[u(\cdot,t-(1-x))](1)\,, & t > 1-x \end{cases}}$$

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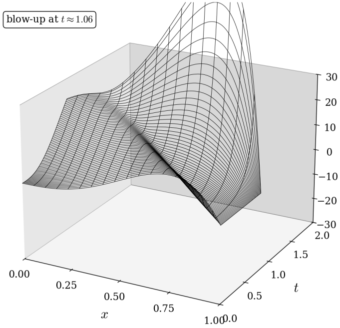
Banach contraction on $C([0,\tau]; L^\infty)$

Simulation

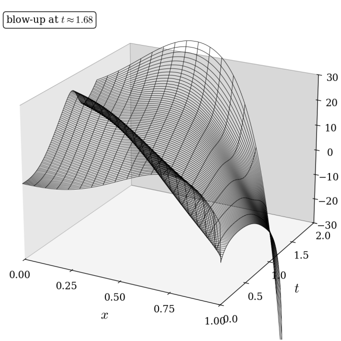
$$u_t = u_x + \frac{1}{2}\left(\int_0^x u\right)^2$$

(a) Open loop, $U \equiv 0$



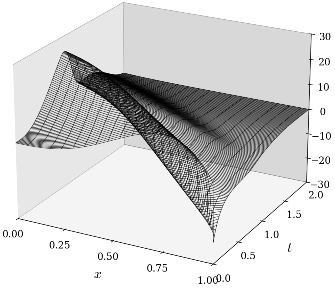
open loop

(b) Second-order feedback, $U = K_2[u](1,t)$



$N = 2$

(c) Third-order feedback, $U = K_2[u](1,t) + K_3[u](1,t)$



$N = 3$

Exact FL

Truncated controller

Neural operator for full feedback law

Adaptive + NO controller

2

The **Machine Learning** Half

Exact FL

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Truncation is finite, but *still* numerically expensive

For each plant F,

solve $N - 1$ kernel PDEs

on simplices up to dimension N

and evaluate an N-fold nested integral

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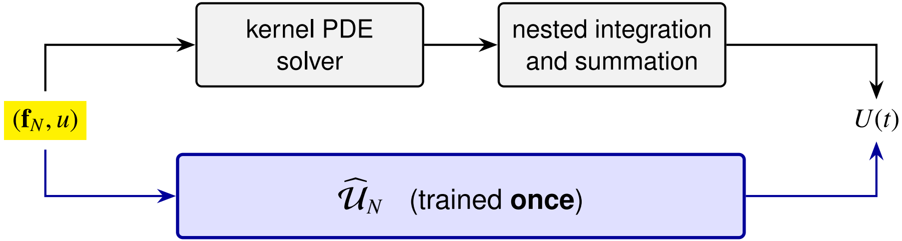
INSTEAD: learn controller map

$$\boxed{\mathcal{U}_N : \quad (\mathbf{f}_N, u) \quad \mapsto \quad K_N[u](1)}$$

Learn $\mathcal{U}_N$ as a neural operator



Lipschitzness

Theorem (Joint Lipschitzness of $\mathcal{U}_N$)

On a class of analytic-bounded plants and an L^∞ ball of states,

$$\left|\mathcal{U}_N(\mathbf{f}_N, u) - \mathcal{U}_N(\tilde{\mathbf{f}}_N, \tilde{u})\right| \;\leq\; \mathcal{L}_f \, \|\mathbf{f}_N - \tilde{\mathbf{f}}_N\|_\star \;+\; \mathcal{L}_u \, \|u - \tilde{u}\|_{L^\infty}$$

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where

$$\boxed{\mathcal{L}_f = \frac{1}{b_f^2} \sum_{n=2}^{N} \frac{(n-1)!}{n} (b_f B_u)^n \qquad \mathcal{L}_u = \frac{B_f}{b_f} \sum_{n=1}^{N-1} n! \, (b_f B_u)^n}$$

and $b_f := \max\{1, 6B_f\}$, $\|f\|_{L^\infty} \le B_f$, $\|u\|_{L^\infty(0,1)} \le B_u$.

Lipschitzness (⟹ neural approximability)

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and $b_f := \max\{1, 6B_f\}$, $\|f\|_{L^\infty} \leq B_f$, $\|u\|_{L^\infty(0,1)} \leq B_u$.

$$\Longrightarrow \boxed{\exists \text{ neural operator } \widehat{\mathcal{U}}_N \text{ with } \left|\mathcal{U}_N - \widehat{\mathcal{U}}_N\right| \leq \varepsilon}$$

Closed loop under $\widehat{\mathcal{U}}_N$

Boundary residual splits:

$$\underbrace{[\widehat{\mathcal{U}}_N - \mathcal{U}_N]}_{\leq \varepsilon \;\; \textit{(training)}} + \underbrace{[\mathcal{U}_N - \mathcal{U}]}_{\leq \varepsilon_N \;\; \textit{(truncation)}}$$

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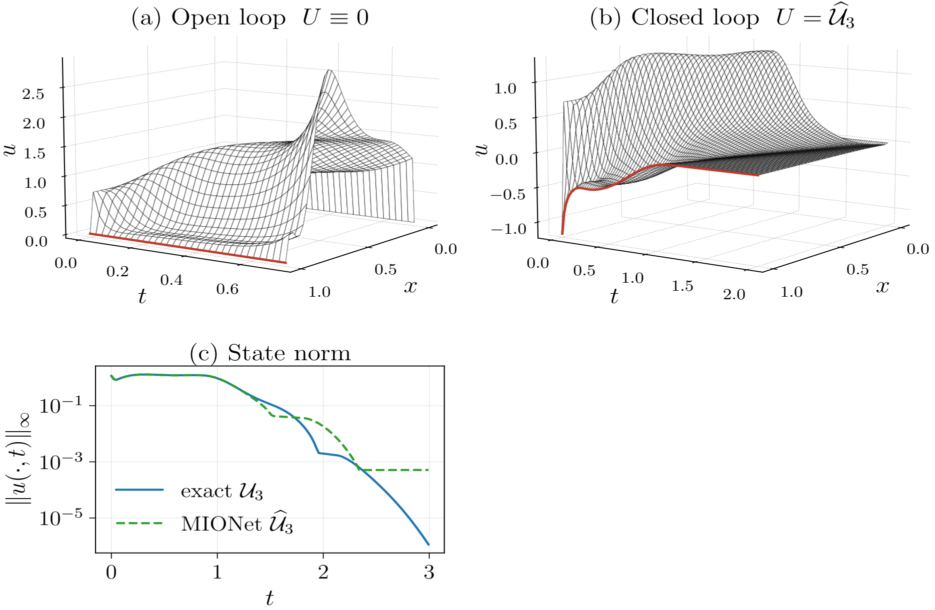
Theorem (Practical stability under NO feedback)

On explicit ball, $\exists \beta \in \mathcal{KL}$ s.t.

$$\|u(\cdot, t)\|_\infty \ \leq \ \beta(\|u_0\|_\infty, t) \ \boxed{+\, \gamma(F, N)\, \varepsilon}$$

Numerical illustration

Luke Bhan



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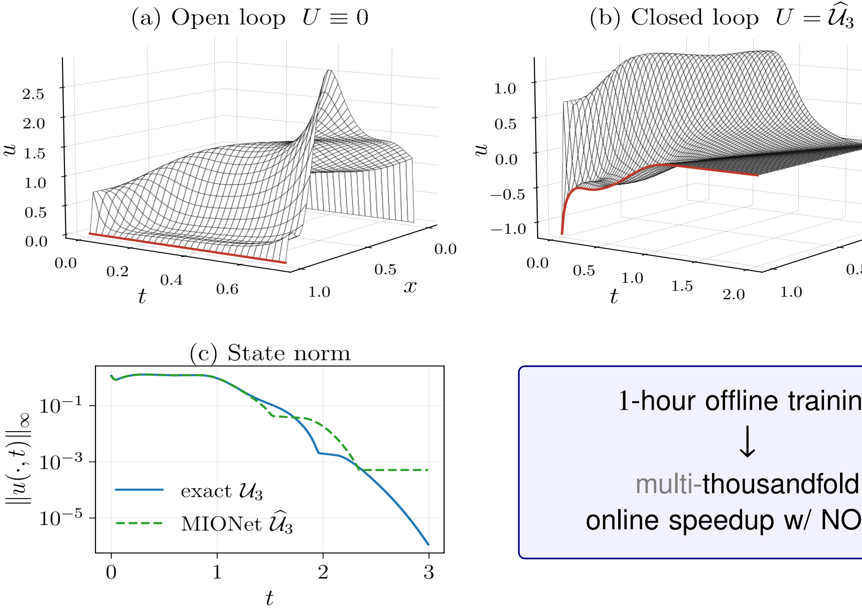

1-hour offline training

↓

multi-thousandfold online speedup w/ NO $\widehat{\mathcal{U}}_3$

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Adaptive + NO controller

What if the plant $\mathbf{f}_N = (f_1, \ldots, f_N)$ is unknown?

adaptive control = learn *plant* **ONLINE**

Observer + parameter update

Observer (state $\hat{u}$, fed by plant state u):

$$\boxed{\hat{u}_t = \hat{u}_x + F_N[u, \hat{\mathbf{f}}] + c_0\,(u - \hat{u})\,\Phi_N(u)\,, \qquad \hat{u}(1,t) = U(t)}$$

nonlinear damping $\Phi_N(u) = \sum_{n=1}^{N} \frac{\gamma_n}{n!}\,\|u\|_{L^2}^{2n}$

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Parameter update (gradient-projection onto $\bar{\mathcal{F}}_n = \{g : \|g\|_\infty \leq n! D_f / \rho_f^{n-1}\}$):

$$\boxed{\partial_t \hat{f}_n = \mathrm{Proj}_{\bar{\mathcal{F}}_n} \left\{ \gamma_n\, e^{cx}\, (u(x,t) - \hat{u}(x,t)) \prod_{i=1}^{n} u(\xi_i, t)\,,\ \hat{f}_n \right\}}$$

What if the plant $\mathbf{f}_N = (f_1, \ldots, f_N)$ is unknown?

adaptive control = learn *plant* **ONLINE**

NO control = learn *mapping to controller gains* **OFFLINE**

Combine the two

Two kernels from one parameter estimate

$$\mathbf{f}_N \xrightarrow{\textit{estimated by}} \hat{\mathbf{f}}_N \xrightarrow[\textit{the kernel PDE for}]{\textit{used to solve } \textbf{exactly}} \check{k}_N$$

ANALYSIS

Two kernels from one parameter estimate

$$\begin{tikzcd}
\mathbf{f}_N \arrow[r, "\textit{estimated by}"] & \hat{\mathbf{f}}_N \arrow[r, "\textit{used to solve } \textbf{exactly} \textit{ the kernel PDE for}"] \arrow[dr, "\textit{plugged into } \widehat{Q}_N \textit{ to compute}"'] & \check{k}_N & \textbf{ANALYSIS} \\
& & \hat{k}_N & \textbf{IMPLEMENTATION}
\end{tikzcd}$$

Two neural operators in this presentation

— full feedback and adaptive

$$(\mathbf{f}_N, u) \longrightarrow \boxed{\widehat{\mathcal{U}}_N} \longrightarrow U$$

Two neural operators in this presentation

— **full feedback** and **adaptive**

$(\mathbf{f}_N, u) \longrightarrow \boxed{\widehat{\mathcal{U}}_N} \longrightarrow U$

$\hat{\mathbf{f}}_N \longrightarrow \boxed{\widehat{Q}_N} \longrightarrow \hat{\mathbf{k}}_N$

Lipschitzness ($\Rightarrow$ neural approximability)

Theorem (Lipschitzness of the kernel-generating map $\mathcal{Q}_N$)

On the projection box of admissible plant estimates,

$$\left\|\mathcal{Q}_N(\hat{\mathbf{f}})_n - \mathcal{Q}_N(\hat{\mathbf{f}}')_n\right\|_{L^2} \leq \ell_n \, \|\hat{\mathbf{f}} - \hat{\mathbf{f}}'\|_\star$$

where

$$\boxed{\ell_n = (n!)^3 \left(12 \max\{1, D_f, 2/\rho_f\}^2\right)^n e^{2n+\frac{8}{3}nD_f^2+2nN(N+1)D_f}}$$

Analysis under FIVE *vanishing* perturbations

Target:

$$\boxed{\begin{array}{rcll} w_t & = & w_x & + \text{ four interior } \textbf{vanishing} \text{ perturbations} \\ w(1,t) & = & \Gamma(t) & \quad \text{boundary } \textbf{vanishing} \text{ perturbation} \end{array}}$$

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Interior:

- R_N^u — *plant Volterra tail*
- R_N^K — *bilinear $\check{k}_n \otimes \hat{f}_m$ cross-orders $n+m-1 > N$*
- $D = c_0\, \Phi_N(u)\, (I - \check{K}_N^*)\, (u - \hat{u})$ — *observer damping*
- $\Omega = \sum_n \int_{T_n(x)} \partial_t \check{k}_n \prod_i \hat{u}\, d\xi$ — *adaptation rate*

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Boundary: $\Gamma(t) = -\sum_n \int_{T_n(1)} (\check{k}_n - \hat{k}_n) \prod_i \hat{u}\, d\xi$ with $\|\check{k}_n - \hat{k}_n\|_{L^2} \leq \varepsilon$

Lyapunov functional (not global, **but robust**)

$$\boxed{V_{\text{tot}} \;=\; \|w\|_c^2 \;+\; \lambda \|u - \hat{u}\|_c^2 \;+\; \lambda \sum_{n=1}^{N} \frac{1}{\gamma_n} \|f_n - \hat{f}_n\|^2_{L^2(T_n(1))}}$$

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No parameter drift, no σ-modification, no PE

CROWN RESULT

Theorem

$\exists$ *NO* $\widehat{Q}_N$ *and* $\bar{\Pi}, \mu_0 > 0$ *s.t.* $\forall$ *initial condition w/* $\Pi(0) \;\leq\; \bar{\Pi}\,,$

- *Lyapunov stability:*

$$\Pi(t) \;\leq\; \mu_0\, \Pi(0)\,, \qquad t \geq 0$$

where

$$\Pi(t) \;=\; \sqrt{\|u(\cdot,t)\|^2_{L^2} \;+\; \|\hat{u}(\cdot,t)\|^2_{L^2} \;+\; \sum_{n=1}^{N} \|f_n - \hat{f}_n(\cdot,t)\|^2_{L^2(T_n(1))}}$$

CROWN RESULT

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$$\Pi(t) \;\leq\; \mu_0\,\Pi(0)\,, \qquad t \geq 0$$

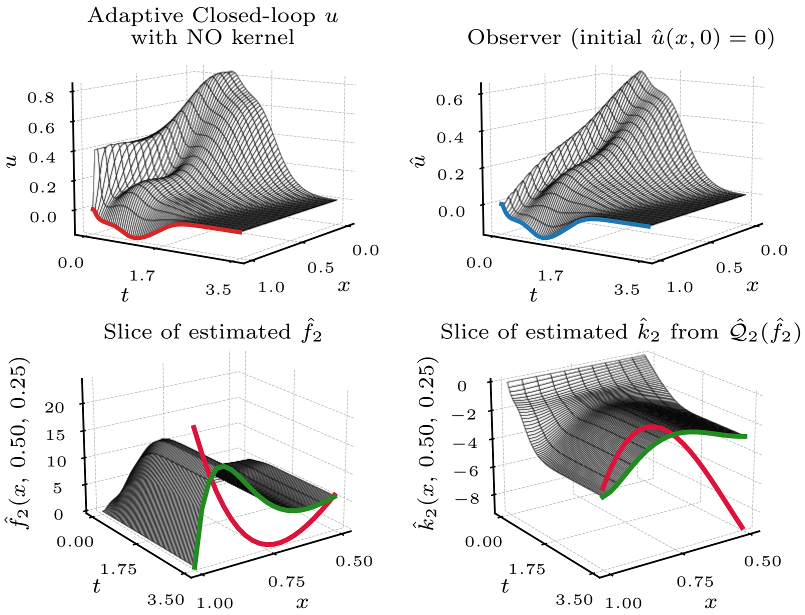
where

$$\Pi(t) \;=\; \sqrt{\|u(\cdot,t)\|^2_{L^2} \;+\; \|\hat{u}(\cdot,t)\|^2_{L^2} \;+\; \sum_{n=1}^{N} \|f_n - \hat{f}_n(\cdot,t)\|^2_{L^2(T_n(1))}}$$

- *Asymptotic regulation:*

$$\|u(\cdot,t)\|_{L^2},\; \|\hat{u}(\cdot,t)\|_{L^2},\; |U(t)| \;\rightarrow\; 0$$

Numerical Illustration: Adaptive $(N = 2)$



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from infinite	to	**finite**
from finite	to	offline **LEARNED**
from offline learned	to	<u>**ONLINE LEARNED**</u>

Questions?